# Equilibrium Default and the Unemployment Accelerator

Julio Blanco University of Michigan

Gaston Navarro Federal Reserve Board

December 2015

These views are those of the authors and not necessarily those of the Board of Governors or the Federal Reserve System.

Julio Blanco and Gaston Navarro "Unemployment Accelerator"

- o Macro-finance models: financial frictions typically affect investment
  - $\Rightarrow$  hard to obtain large contractions in output and employment

- o Macro-finance models: financial frictions typically affect investment
  - $\Rightarrow$  hard to obtain large contractions in output and employment

 Recent work: link between financial conditions and labor costs (Jermann & Quadrini, 2012), (Monacelli, Quadrini & Trigari, 2014), (Petrosky-Nadeau, 2014)

financial conditions  $\Rightarrow$  labor market

- o Macro-finance models: financial frictions typically affect investment
  - $\Rightarrow$  hard to obtain large contractions in output and employment
- Recent work: link between financial conditions and labor costs (Jermann & Quadrini, 2012), (Monacelli, Quadrini & Trigari, 2014), (Petrosky-Nadeau, 2014)

financial conditions  $\Rightarrow$  labor market

o This paper: labor is an asset for the firm

- o Macro-finance models: financial frictions typically affect investment
  - $\Rightarrow$  hard to obtain large contractions in output and employment
- Recent work: link between financial conditions and labor costs (Jermann & Quadrini, 2012), (Monacelli, Quadrini & Trigari, 2014), (Petrosky-Nadeau, 2014)

financial conditions  $\Rightarrow$  labor market

- o This paper: labor is an asset for the firm
  - + link labor market and financial conditions
  - + value of a worker affects financial conditions!

labor market  $\Leftrightarrow$  financial conditions

- o Macro-finance models: financial frictions typically affect investment
  - $\Rightarrow$  hard to obtain large contractions in output and employment
- Recent work: link between financial conditions and labor costs (Jermann & Quadrini, 2012), (Monacelli, Quadrini & Trigari, 2014), (Petrosky-Nadeau, 2014)

financial conditions  $\Rightarrow$  labor market

- o This paper: labor is an asset for the firm
  - + link labor market and financial conditions
  - + value of a worker affects financial conditions!

labor market  $\Leftrightarrow$  financial conditions

o labor market accounts for a large fraction of financial market fluctuations

• Firm with productivity x, wage w, n workers and debt  $\overline{b}$ .

- Firm with productivity x, wage w, n workers and debt  $\overline{b}$ .
- o Search friction: x > w (Pissarides, 1985)
- o Value of a worker

$$S = \underbrace{(x - w)}_{>0} + \beta S' > 0$$

- Firm with productivity x, wage w, n workers and debt  $\overline{b}$ .
- o Search friction: x > w (Pissarides, 1985)
- o Value of a worker

$$S = \underbrace{(x - w)}_{>0} + \beta S' > 0$$

o Option to default



- Firm with productivity x, wage w, n workers and debt  $\overline{b}$ .
- o Search friction: x > w (Pissarides, 1985)
- o Value of a worker

$$S = \underbrace{(x - w)}_{>0} + \beta S' > 0$$

o Option to default



o Main mechanism

 $\Downarrow S \Rightarrow \uparrow \mathsf{Default} \Rightarrow \uparrow \mathsf{Borrowing costs} \Rightarrow \Downarrow S$ 

## Findings

#### Model:

- o Fluctuations in the labor market
  - + explain 68% of credit spreads volatility ....
  - +  $\ldots$  and 80% of default rates volatility.

#### Evidence: (very preliminary!)

- o A 10% decline in employment volatility
  - + associated with a 4% decline in default rates volatility...

# Model

#### Environment

- o Demography: Family with measure one of workers, and firms.
- Preferences:  $\mathbb{E}_0[\sum_{t=0}^{\infty} \beta^t C_t]$ .
- **Technology**:  $y_i = xa_i n_i$ .
- Shocks: a<sub>i</sub> ∼ H i.i.d. across firms and time; x follow a Markov process.
- o Labor Market: search friction as in of Mortensen-Pissarides (1994).
- o Capital Market: firms' debt, subject to default risk.
  - + Upon **default**, a firm disappears.
  - + Long-term debt: a fraction  $\lambda$  matures every period.



Figure: Timing of events in period t

## Firm's problem

$$E(a, \bar{b}, n, z) = \max_{d, nd} \left\{ 0, \max_{\mu = \{d, \bar{v}, \bar{b}'\}} \left\{ d + \beta \mathbb{E}_{z', a'} \left[ E(a', \bar{b}', n', z') | z \right] \right\} \right\}$$
  
subject to  
$$y = axn - w(a, b, z, \mu)n - (1 - \tau)\lambda \bar{b}$$
$$d + \kappa \bar{v} \leq y + p(b', z) \underbrace{\left[ \bar{b}' - (1 - \lambda) \bar{b} \right]}_{n'}$$
$$n' = (1 - s)n + \bar{v}q(z)$$
with  $b' = \bar{b}'/n'$ 

a = firm's productivity,  $\overline{b} =$  debt, n = workers, z = aggregate state  $\overline{v} =$  vacancies, q = prob of filling a vacancy

## Firms cont'd

#### Lemma

Firm's value function is linear:  $E(a, \overline{b}, n, z) = e(a, b, z)n$ .

Policies are linear in n and independent of y

$$ar{m{v}}(a,ar{b},n,z) = m{v}(b,z)n$$
  
 $ar{b}'(a,ar{b},n,z) = m{b}'(b,z)n$ 

Default follows a productivity threshold decision  $\underline{a}(b, z)$ :

$$\begin{cases} default & \text{if } a \leq \underline{a}(b, z) \\ no \ default & \text{if } a \leq \underline{a}(b, z) \end{cases}$$

## Firms cont'd

#### Lemma

Firm's value function is linear:  $E(a, \overline{b}, n, z) = e(a, b, z)n$ .

Policies are linear in n and independent of y

$$ar{m{v}}(a,ar{b},n,z) = m{v}(b,z)n$$
  
 $ar{b}'(a,ar{b},n,z) = m{b}'(b,z)n$ 

Default follows a productivity threshold decision  $\underline{a}(b, z)$ :

$$\begin{cases} default & \text{if } a \leq \underline{a}(b, z) \\ no \ default & \text{if } a \leq \underline{a}(b, z) \end{cases}$$

**Notation**: Let b denote a firm's debt per worker and B the average debt per worker over firms.

Let  $\mathcal{W}(a, b, z)$  and  $\mathcal{U}(z)$  be the vale of being employed and unemployed.

Let  $\mathcal{W}(a, b, z)$  and  $\mathcal{U}(z)$  be the vale of being employed and unemployed.

• Value of unemployment  $\mathcal{U}(z)$ 

 $\mathcal{U}(z) = \bar{u} + \beta \mathbb{E}_{a',z'} \left[ f(z) \mathcal{W}(a', B(z'), z') + (1 - f(z)) \mathcal{U}(z') | z \right]$ 

#### Workers

Let  $\mathcal{W}(a, b, z)$  and  $\mathcal{U}(z)$  be the vale of being employed and unemployed.

• Value of unemployment  $\mathcal{U}(z)$ 

$$\mathcal{U}(z) = \bar{u} + \beta \mathbb{E}_{a',z'} \left[ f(z) \mathcal{W}(a', B(z'), z') + (1 - f(z)) \mathcal{U}(z') | z \right]$$

o If  $a > \underline{a}(b, z)$ , value of being employed  $\mathcal{W}(a, b, z)$ 

$$\mathcal{W}(a, b, z) = w(a, b, z) + \beta \mathbb{E}_{a', z'} \left[ (1 - s) \mathcal{W}(a', \boldsymbol{b}'(b, z), z') + s \mathcal{U}(z') | z \right]$$

#### Workers

Let  $\mathcal{W}(a, b, z)$  and  $\mathcal{U}(z)$  be the vale of being employed and unemployed.

• Value of unemployment  $\mathcal{U}(z)$ 

$$\mathcal{U}(z) = \bar{u} + \beta \mathbb{E}_{a',z'} \left[ f(z) \mathcal{W}(a', B(z'), z') + (1 - f(z)) \mathcal{U}(z') | z \right]$$

o If  $a > \underline{a}(b, z)$ , value of being employed  $\mathcal{W}(a, b, z)$ 

$$\mathcal{W}(a, b, z) = w(a, b, z) + \beta \mathbb{E}_{a', z'} \left[ (1 - s) \mathcal{W}(a', b'(b, z), z') + s \mathcal{U}(z') | z \right]$$

o If 
$$a \leq \underline{a}(b, z)$$
,  $\mathcal{W}(a, b, z) = \mathcal{U}(z)$ 

Let  $\mathcal{W}(a, b, z)$  and  $\mathcal{U}(z)$  be the vale of being employed and unemployed.

• Value of unemployment  $\mathcal{U}(z)$ 

$$\mathcal{U}(z) = \bar{u} + \beta \mathbb{E}_{a',z'} \left[ f(z) \mathcal{W}(a', B(z'), z') + (1 - f(z)) \mathcal{U}(z') | z \right]$$

o If  $a > \underline{a}(b, z)$ , value of being employed  $\mathcal{W}(a, b, z)$ 

$$\mathcal{W}(a, b, z) = w(a, b, z) + \beta \mathbb{E}_{a', z'} \left[ (1 - s) \mathcal{W}(a', \boldsymbol{b}'(b, z), z') + s \mathcal{U}(z') | z \right]$$

o If 
$$a \leq \underline{a}(b, z)$$
,  $\mathcal{W}(a, b, z) = \mathcal{U}(z)$ 

o Let  $g(a, b, z) = \mathcal{W}(a, b, z) - \mathcal{U}(z)$  be the worker's surplus.

Let  $\mathcal{W}(a, b, z)$  and  $\mathcal{U}(z)$  be the vale of being employed and unemployed.

• Value of unemployment  $\mathcal{U}(z)$ 

$$\mathcal{U}(z) = \bar{u} + \beta \mathbb{E}_{a',z'} \left[ f(z) \mathcal{W}(a', B(z'), z') + (1 - f(z)) \mathcal{U}(z') | z \right]$$

o If  $a > \underline{a}(b, z)$ , value of being employed  $\mathcal{W}(a, b, z)$ 

$$\mathcal{W}(a, b, z) = w(a, b, z) + \beta \mathbb{E}_{a', z'} \left[ (1 - s) \mathcal{W}(a', \boldsymbol{b}'(b, z), z') + s \mathcal{U}(z') | z \right]$$

o If 
$$a \leq \underline{a}(b,z)$$
,  $\mathcal{W}(a,b,z) = \mathcal{U}(z)$ 

• Let  $g(a, b, z) = \mathcal{W}(a, b, z) - \mathcal{U}(z)$  be the worker's surplus.

#### Wages

Wages given by Nash bargaining.

more

•  $\gamma$  firm's bargaining power.

## Aggregates

o Matching function

$$f = \frac{m(V, U)}{U}$$
 and  $q = \frac{m(V, U)}{V}$ 

o Unemployment

$$U(z) = (1 - N) + \underbrace{H(\underline{a}(b, z))}_{\text{endorenous separation}} N$$

o Law of motion for labor

$$N' = (1-s)\left[1 - H(\underline{a}(b,z))\right]N + f(z)U(z)$$

• State of the economy z = (x, B, N).

Equilibrium Definition

# Model Characterization

#### Assumption

- 1. One period debt  $\lambda = 1$ .
- 2. A Cobb-Douglas matching function:  $m(U, V) = U^{1-\nu} V^{\nu}$ .

• Let S(a, b, z) = e(a, b, z) + g(a, b, z) be the surplus of a match

o Let S(a, b, z) = e(a, b, z) + g(a, b, z) be the surplus of a match

#### Proposition

For a given firm policies - b', v and  $\underline{a}$ - the surplus of a match is

$$= \left\{ \begin{array}{l} 0, \quad \left\{ ax - \bar{u} - \kappa v \right. \\ - \underbrace{(1 - \tau)b + p(b', z)b'\left[(1 - s) + q(z)v\right]}_{\text{Debt outflow}} \\ + \underbrace{(1 - s)\beta \mathbb{E}_{a',z'}\left[S(a', b', z')|z\right]}_{\text{continuation value of a match}} \\ + \underbrace{\beta \mathbb{E}_{a',z'}\left[q(z)v\gamma S(a', b', z') - \underbrace{f(z)(1 - \gamma)S(a', B(z'), z')}_{\text{Workers outside value}}|z\right]} \right\} \right\}$$

o Let S(a, b, z) = e(a, b, z) + g(a, b, z) be the surplus of a match

#### Proposition

For a given firm policies - b', v and  $\underline{a}$ - the surplus of a match is

$$= \max_{d,nd} \left\{ 0, \max_{v,b'} \left\{ ax - \overline{u} - \kappa v \right. \right. \\ \left. - \underbrace{(1-\tau)b + p(b',z)b' \left[ (1-s) + q(z)v \right]}_{Debt outflow} \\ + \left. (1-s)\beta \underbrace{\mathbb{E}_{a',z'} \left[ S(a',b',z') | z \right]}_{continuation value of a match} \\ + \beta \underbrace{\mathbb{E}_{a',z'} \left[ q(z)v\gamma S(a',b',z') - \underbrace{f(z)(1-\gamma)S(a',B(z'),z')}_{Workers outside value} | z \right]}_{Firm's growth} \right\}$$

o Let S(a, b, z) = e(a, b, z) + g(a, b, z) be the surplus of a match

#### Proposition

Firm's policies -  $b'(\cdot)$ ,  $v(\cdot)$  and  $\underline{a}(\cdot)$  - maximize the the joint surplus, and solve

$$S(a, b, z) = \max_{d, nd} \left\{ 0, \max_{v, b'} \left\{ ax - \bar{u} - \kappa v - \underbrace{(1 - \tau)b + p(b', z)b'\left[(1 - s) + q(z)v\right]}_{\text{Debt outflow}} + (1 - s)\beta \underbrace{\mathbb{E}_{a', z'}\left[S(a', b', z')|z\right]}_{\text{continuation value of a match}} + \beta \mathbb{E}_{a', z'}\left[\underbrace{q(z)v\gamma S(a', b', z')}_{\text{Firm's growth}} - \underbrace{f(z)(1 - \gamma)S(a', B(z'), z')}_{\text{Workers outside value}}|z|\right\} \right\}$$

o Let S(a, b, z) = e(a, b, z) + g(a, b, z) be the surplus of a match

#### Proposition

Firm's policies -  $b'(\cdot)$ ,  $v(\cdot)$  and  $\underline{a}(\cdot)$  - maximize the the joint surplus, and solve

$$S(a, b, z) = \max_{d, nd} \left\{ 0, \max_{v, b'} \left\{ ax - \bar{u} - \kappa v - \underbrace{(1 - \tau)b + p(b', z)b'\left[(1 - s) + q(z)v\right]}_{\text{Debt outflow}} + (1 - s)\beta \underbrace{\mathbb{E}_{a', z'}\left[S(a', b', z')|z\right]}_{\text{continuation value of a match}} + \beta \mathbb{E}_{a', z'} \underbrace{\left[q(z)v\gamma S(a', b', z') - \underbrace{f(z)(1 - \gamma)S(a', B(z'), z')}_{\text{Workers outside value}}\right]z\right\}}$$

+ Only equation to solve in the model!

Exercise

In equilibrium, default threshold is given as

$$\underline{a}(b,z) = \frac{1}{x} \left[ \overbrace{(1-\tau)b - p(b'(\cdot), z)b'(\cdot)(1-s)}^{\text{Debt outflow}} - \overline{u} \dots - (1-s - f(z)(1-\gamma))\beta \underbrace{\mathbb{E}_{a',z'}\left[S(a', b'(\cdot), z')|z\right]}_{\text{Value of a match}} \right]$$

+ Financial market component: Debt *b* and bond price  $p(b'(\cdot), z)$ . + Labor market component: Joint surplus  $S(a', b'(\cdot), z')$ 

Exercise

In equilibrium, default threshold is given as

$$\underline{a}(b,z) = \frac{1}{x} \left[ \overbrace{(1-\tau)b - p(b'(\cdot), z)b'(\cdot)(1-s)}^{\text{Debt outflow}} - \overline{u} \dots - (1-s - f(z)(1-\gamma))\beta \underbrace{\mathbb{E}_{a',z'}\left[S(a', b'(\cdot), z')|z\right]}_{\text{Value of a match}} \right]$$

+ Financial market component: Debt *b* and bond price  $p(b'(\cdot), z)$ . + Labor market component: Joint surplus  $S(a', b'(\cdot), z')$ 

Worsening in the value of a match  $\Rightarrow$  Worsening in financial conditions

## Financial value of a worker

• Value of a match if  $a > \underline{a}(b, z)$ 

$$S(a, b, z) = \overbrace{xa - \overline{u} + (1 - s + f(z)(1 - \gamma)) \mathbb{E}_{a', z'} \left[ S(a', b'(\cdot), z') | z \right]}^{\text{Standard DMP}} \dots$$

$$- \underbrace{(1-\tau)b + p(\boldsymbol{b}'(\cdot), z)\boldsymbol{b}'(\cdot)\left[(1-s) + q(z)\boldsymbol{\nu}(\cdot)\right]}_{(1-s)+(1-$$

Debt outflow

#### Financial value of a worker

• Value of a match if  $a > \underline{a}(b, z)$ 

$$S(a, b, z) = \underbrace{xa - \overline{u} + (1 - s + f(z)(1 - \gamma)) \mathbb{E}_{a', z'} \left[ S(a', b'(\cdot), z') | z \right]}_{- (1 - \tau)b + p(b'(\cdot), z)b'(\cdot) \left[ (1 - s) + q(z)v(\cdot) \right]}$$

Debt outflow

Free entry condition

$$\kappa \underbrace{f(z)^{\frac{1-\nu}{\nu}}}_{1/q(z)} = \underbrace{p(\mathbf{b}'(\cdot), z)\mathbf{b}'(\cdot)}_{\text{Financial value}} + \underbrace{\gamma\beta\mathbb{E}_{a', z'}\left[S(a', \mathbf{b}'(\cdot), z')|z\right]}_{\text{Standard DMP}}$$

#### Financial value of a worker

• Value of a match if  $a > \underline{a}(b, z)$ 

$$S(a, b, z) = \underbrace{xa - \overline{u} + (1 - s + f(z)(1 - \gamma)) \mathbb{E}_{a', z'} \left[ S(a', b'(\cdot), z') | z \right]}_{\text{Debt outflow}} \dots$$

Free entry condition



Worsening in financial conditions  $\Rightarrow$  Worsening in the value of a match












## Model Evaluation

#### o Institutional parameters

- + Tax benefit au = 13% U.S. Government Accountability Office (2013)
- + Maturity  $\lambda=1/24$  average maturity of 2 years

more

#### o Institutional parameters

- + Tax benefit au = 13% U.S. Government Accountability Office (2013)
- + Maturity  $\lambda = 1/24$  average maturity of 2 years
- o Technology
  - + Porductivity In  $a \sim N(-\frac{1}{2}\sigma_a^2, \sigma_a^2)$  with  $\sigma_a = 0.2$ , match annual default of 1%.
  - + Aggregate productivity:  $\ln x \sim AR(1)$  with  $(\rho_x, \sigma_x) = (0.98, 0.005)$ .

- o Institutional parameters
  - +~ Tax benefit  $\tau =$  13% U.S. Government Accountability Office (2013)
  - + Maturity  $\lambda = 1/24$  average maturity of 2 years
- Technology
  - + Porductivity ln  $a \sim N(-\frac{1}{2}\sigma_a^2, \sigma_a^2)$  with  $\sigma_a = 0.2$ , match annual default of 1%.
  - + Aggregate productivity:  $\ln x \sim AR(1)$  with  $(\rho_x, \sigma_x) = (0.98, 0.005)$ .
- o Labor market:
  - + Unemployment benefit  $\bar{u} =$  0.6, Shimer (2005) and Hagedorn-Manovskii (2008)

- o Institutional parameters
  - +~ Tax benefit  $\tau =$  13% U.S. Government Accountability Office (2013)
  - + Maturity  $\lambda = 1/24$  average maturity of 2 years
- Technology
  - + Porductivity ln  $a \sim N(-\frac{1}{2}\sigma_a^2, \sigma_a^2)$  with  $\sigma_a = 0.2$ , match annual default of 1%.
  - + Aggregate productivity:  $\ln x \sim AR(1)$  with  $(\rho_x, \sigma_x) = (0.98, 0.005)$ .
- o Labor market:
  - + Unemployment benefit  $\bar{u} =$  0.6, Shimer (2005) and Hagedorn-Manovskii (2008)
- o Global solution piecewise linear approximation.

	Std. dev		Corr w/ Output	
	Data	Model	Data	Model
Finding	0.13	0.08	0.87	0.63
Unemployment	0.19	0.11	-0.88	-0.63
Credit Spreads	0.62	0.41	-0.46	-0.54
Default Rate	0.20	1.67	-0.31	-0.48

Note: Data is monthly for the period 1951-2012. Model correlations and standard deviations are computed as average over 50,000 independently simulated economies, of 61 years of length. All variables are in log deviation from an HP trend with smoothing parameter of  $10^5$ .

	Std. dev		Corr w/ Output	
	Data	Model	Data	Model
Finding	0.13	0.08	0.87	0.63
Unemployment	0.19	0.11	-0.88	-0.63
Credit Spreads	0.62	0.41	-0.46	-0.54
Default Rate	0.20	1.67	-0.31	-0.48

Note: Data is monthly for the period 1951-2012. Model correlations and standard deviations are computed as average over 50,000 independently simulated economies, of 61 years of length. All variables are in log deviation from an HP trend with smoothing parameter of  $10^5$ .

	Std. dev		Corr w/ Output	
	Data	Model	Data	Model
Finding	0.13	0.08	0.87	0.63
Unemployment	0.19	0.11	-0.88	-0.63
Credit Spreads	0.62	0.41	-0.46	-0.54
Default Rate	0.20	1.67	-0.31	-0.48

Note: Data is monthly for the period 1951-2012. Model correlations and standard deviations are computed as average over 50,000 independently simulated economies, of 61 years of length. All variables are in log deviation from an HP trend with smoothing parameter of  $10^5$ .

### Model Response to a productivity shock



#### Figure: Model Response to a productivity shock

Note: Impulse responses correspond to the average over 50,000 independently simulated economies, all with the same productivity innovation at t = 0.

### Evaluating the mechanism

What is the effect of movements in the labor market on the financial market?

#### Exercise:

• Default threshold depends on f(z) and S(a, B, z).

#### Exercise:

• Default threshold depends on f(z) and S(a, B, z).

nore

- o Endow the government with two instruments
  - + A transfer to firms T(z).
  - + A vacancy posting subsidy  $T^{\nu}(z)$ .

#### Exercise:

- Default threshold depends on f(z) and S(a, B, z).
- o Endow the government with two instruments
  - + A transfer to firms T(z).
  - + A vacancy posting subsidy  $T^{\nu}(z)$ .
- Set T(z) and  $T^{v}(z)$  such that

$$f(z)=f^*$$
 and  $\mathbb{E}_a\left[S(a,B,z)
ight]=S^*$ 

#### Exercise:

- Default threshold depends on f(z) and S(a, B, z).
- o Endow the government with two instruments
  - + A transfer to firms T(z).
  - + A vacancy posting subsidy  $T^{\nu}(z)$ .
- Set T(z) and  $T^{v}(z)$  such that

$$f(z) = f^*$$
 and  $\mathbb{E}_a[S(a, B, z)] = S^*$ 

o Compare business cycle statistics in both models.

### Evaluating the mechanism

	Data	Full Model	Fix $f$ and $S$
Finding	0.13	0.08	-
Unemployment	0.19	0.11	0.004
Credit Spreads	0.62	0.41	0.13
Default Rate	0.20	1.67	0.36

**Note:** Model is mean and standard deviation over 50,000 bootstrap simulations with simulation length of 61 years. All variables are log as deviation from an HP trend with smoothing parameter 10<sup>5</sup>.

+ Labor market accounts for 68% and 80% of credit spreads and default fluctuations rate, respectively

# Evidence (very, very, very preliminary ...)

- o Link between volatility of labor market and firms' default risk ....
- o ceteris paribus: more volatile labor markets  $\Rightarrow$  more volatile dfault risk

- o Link between volatility of labor market and firms' default risk ....
- o ceteris paribus: more volatile labor markets  $\Rightarrow$  more volatile dfault risk

#### Idea:

o Compute a measure of firms' default risk

+ Distance to Default

Merton's mode

DD Graph

(Gilchrist & Zakrajsek, 2009), (Duffie, 2009)

- o Link between volatility of labor market and firms' default risk ....
- o ceteris paribus: more volatile labor markets  $\Rightarrow$  more volatile dfault risk

#### Idea:

- o Compute a measure of firms' default risk
  - + Distance to Default Merton's model DI

(Gilchrist & Zakrajsek, 2009), (Duffie, 2009)

o Control default risk by factors other than employment.

- o Link between volatility of labor market and firms' default risk ....
- o ceteris paribus: more volatile labor markets  $\Rightarrow$  more volatile dfault risk

#### Idea:

- o Compute a measure of firms' default risk
  - + Distance to Default Merton's model DD Gra

(Gilchrist & Zakrajsek, 2009), (Duffie, 2009)

- o Control default risk by factors other than employment.
- o Sectoral volatility of employment and default risk (residual).

- Firm *i*, in sector *s*, at quarter *t*.
- o Distance to default  $DD_{it}$ , and prob of default  $\Phi_{it} = \Phi(-DD_{it})$

- Firm *i*, in sector *s*, at quarter *t*.
- Distance to default  $DD_{it}$ , and prob of default  $\Phi_{it} = \Phi(-DD_{it})$

o Control  $\Phi_{it}$  (or  $DD_{it}$ ) on many characteristics, but not employment

$$\Phi_{it} = \gamma_t^{\Phi} + \alpha_i^{\Phi} + X_{it}\beta + \epsilon_{it}^{\Phi}$$

 $X_{it}$ : assets, liabilities, investment, profits/assets, assets/market value

- Firm *i*, in sector *s*, at quarter *t*.
- Distance to default  $DD_{it}$ , and prob of default  $\Phi_{it} = \Phi(-DD_{it})$
- o Control  $\Phi_{it}$  (or  $DD_{it}$ ) on many characteristics, but not employment

$$\Phi_{it} = \gamma_t^{\Phi} + \alpha_i^{\Phi} + X_{it}\beta + \epsilon_{it}^{\Phi}$$

 $X_{it}$ : assets, liabilities, investment, profits/assets, assets/market value

• Employment growth by sector  $\Delta E_{st}$ 

$$\Delta E_{st} = \gamma_t^{\Delta E} + \gamma_s^{\Delta E} + profits_{st}\beta + \epsilon_{st}^{\Delta E}$$

- Firm *i*, in sector *s*, at quarter *t*.
- Distance to default  $DD_{it}$ , and prob of default  $\Phi_{it} = \Phi(-DD_{it})$
- o Control  $\Phi_{it}$  (or  $DD_{it}$ ) on many characteristics, but not employment

$$\Phi_{it} = \gamma_t^{\Phi} + \alpha_i^{\Phi} + X_{it}\beta + \epsilon_{it}^{\Phi}$$

 $X_{it}$ : assets, liabilities, investment, profits/assets, assets/market value

o Employment growth by sector  $\Delta E_{st}$ 

$$\Delta E_{st} = \gamma_t^{\Delta E} + \gamma_s^{\Delta E} + \textit{profits}_{st}\beta + \epsilon_{st}^{\Delta E}$$

o Variances by sector:

$$\sigma_{s}^{2,\Phi} = \frac{1}{TN_{s}} \sum_{t,i \in s} \left( \epsilon_{it}^{\Phi} - \overline{\epsilon}_{s}^{\Phi} \right)^{2} \text{ and } \sigma_{s}^{2,\Delta E} = \frac{1}{TN_{s}} \sum_{t} \left( \epsilon_{st}^{\Delta E} - \overline{\epsilon}_{s}^{\Delta E} \right)^{2}$$

### More volatile labor market, more volatile default risk



Note: Non-financial corporate sector, period 1975q1 - 2014q4. STD on equation residuals. Data source CRSP-Compustat merged panel for 15,320 firms, 372 sectors.

Julio Blanco and Gaston Navarro "Unemployment Accelerator"

#### **Conclusions:**

- o Proposed an interaction between labor and financial markets ....
- o ... "Unemployment accelerator" loop.
- o Important amplification of financial conditions.

#### Conclusions:

- o Proposed an interaction between labor and financial markets ....
- o ... "Unemployment accelerator" loop.
- o Important amplification of financial conditions.

#### Next Steps:

- o Evaluate the mechanism in a larger DSGE model
- o Evidence: need more work ...

#### **Conclusions:**

- o Proposed an interaction between labor and financial markets ....
- o ... "Unemployment accelerator" loop.
- o Important amplification of financial conditions.

#### Next Steps:

- o Evaluate the mechanism in a larger DSGE model
- o Evidence: need more work ...

## Thank you!!!

# Appendix

Return

$$V(\omega, z) = \max_{\{C, b'_h(b'), \omega'\}} \{U(C) + \beta \mathbb{E}_{z'}[V(\omega', z')|z]\}$$

subject to

$$C + \int p(b',z)b'_{h}(b')db' + \mathbb{T}(z) \leq \omega + d(z) + \bar{u}U(z)$$
  
+  $\mathbb{E}_{a}\left[\mathbb{I}\left\{a \geq \underline{a}(B(z),z)\right\}w(a,B(z),z)\right]\bar{N}$   
$$\omega' = \int \left[1 - H(\underline{a}(b',z'))\right]\left[\lambda + (1-\lambda)p(b'(b',z'),z')\right]b'_{h}(b')db'$$
  
$$z' = \Gamma(z)$$

where d(z) are firms' dividend payments
Return

Wages are given by

$$w(a,b,z,v,b') = \arg\max_{\vec{w}} \left\{ \tilde{e}(a,b,z,v,b')_w^\gamma \tilde{g}(a,b,z,v,b')_w^{1-\gamma} \right\}$$

where

$$\begin{array}{lll} \tilde{e}(a,b,z,v,b')_w &=& ax - w - \kappa v - (1-\tau)\lambda b \\ &+& p(b',z) \left[ b'[(1-s) + vq(z)] - (1-\lambda)b \right] \\ &+& \left[ (1-s) + vq(z) \right] \beta \mathbb{E}_{a',z'} \left[ e(a',b',z') | z \right] \end{array}$$

$$\begin{split} \tilde{g}(a, b, z, b')_w &= w - \bar{u} + \\ &+ \beta \mathbb{E}_{a', z'} \left[ (1 - s)g(a', b', z') - f(z)g(a', B(z'), z') | z \right] \\ \text{and } \vec{w} &= \{ w(a, b, z, b') \} \end{split}$$

#### Definition

A recursive equilibrium is given by value functions:  $\{E, W, U, V\}$ ; policies for the firm  $\{\bar{b}', \bar{v}, d\}$ , the household  $\{C, b'_h\}$ ; probabilities  $\{f, q\}$ ; and prices  $\{p(b'), w\}$  such that

- Agents optimize and achieve values E, W, U, and V.
- Wages w solve the Nash bargaining problem.
- (Walrasian) markets clear:
  - + Bonds market:  $\int b'(a, \bar{b}, n, z) = b_h(b', z)$
  - + Goods market:  $Y(z) = C(z) + \int \bar{v}(a, \bar{b}, n, z)$



• Let S(a, b, z) = e(a, b, z) + g(a, b, z) be the joint surplus of a match.

$$\begin{split} S(a, b, z) &= \mathbb{I}_{\{a \leq \underline{a}(\cdot)\}} 0 + \mathbb{I}_{\{a > \underline{a}(\cdot)\}} \left\{ ax - \overline{u} - \kappa \mathbf{v}(\cdot) \right. \\ &- \underbrace{(1 - \tau)b + p(b'(\cdot), z)b'(\cdot)\left[(1 - s) + q(z)\mathbf{v}(\cdot)\right]}_{\text{Debt outflow}} \\ &+ \underbrace{\left[(1 - s) + q(z)\mathbf{v}(\cdot)\right]}_{\text{Firm's continuation value}} \\ &+ \underbrace{\beta \mathbb{E}_{z', a'}\left[(1 - s)g(a', b'(\cdot), z') - f(z)g(a', B(z'), z')|z\right]}_{\text{Firm's continuation value}} \right\} \end{split}$$

Worker's continuation value

$$\begin{aligned} S(a, b, z) &= \mathbb{I}_{\{a \leq \underline{a}(\cdot)\}} 0 + \mathbb{I}_{\{a > \underline{a}(\cdot)\}} \Big\{ ax - \overline{u} - \kappa \mathbf{v}(\cdot) \\ &- (1 - \tau)b + p(b'(\cdot), z)b'(\cdot) \left[ (1 - s) + q(z)\mathbf{v}(\cdot) \right] \\ &+ \left[ (1 - s) + q(z)\mathbf{v}(\cdot) \right] \beta \mathbb{E}_{z', a'} \left[ e(a', b'(\cdot), z') | z \right] \\ &+ \beta \mathbb{E}_{z', a'} \left[ (1 - s)g(a', b'(\cdot), z') - f(z)g(a', B(z'), z') | z \right] \end{aligned}$$

Return

$$\begin{split} S(a, b, z) &= \mathbb{I}_{\{a \leq \underline{a}(\cdot)\}} 0 + \mathbb{I}_{\{a > \underline{a}(\cdot)\}} \Big\{ ax - \overline{u} - \kappa \mathbf{v}(\cdot) \\ &- (1 - \tau)b + p(\mathbf{b}'(\cdot), z)\mathbf{b}'(\cdot) \left[ (1 - s) + q(z)\mathbf{v}(\cdot) \right] \\ &+ (1 - s)\beta \mathbb{E}_{a', z'} \left[ S(a', \mathbf{b}'(\cdot), z') | z \right] \\ &+ \beta \mathbb{E}_{a', z'} \left[ q(z)\mathbf{v}(\cdot)e(a', \mathbf{b}'(\cdot), z') - f(z)g(a', B(z'), z') | z \right] \Big\} \end{split}$$

Return

$$\begin{aligned} S(a, b, z) &= \mathbb{I}_{\{a \leq \underline{a}(\cdot)\}} 0 + \mathbb{I}_{\{a > \underline{a}(\cdot)\}} \Big\{ ax - \overline{u} - \kappa v(\cdot) \\ &- (1 - \tau)b + p(b'(\cdot), z)b'(\cdot) [(1 - s) + q(z)v(\cdot)] \\ &+ (1 - s)\beta \mathbb{E}_{a', z'} \left[ S(a', b'(\cdot), z') | z \right] \\ &+ \beta \mathbb{E}_{a', z'} \left[ q(z)v(\cdot)e(a', b'(\cdot), z') - f(z)g(a', B(z'), z') | z \right] \Big\} \end{aligned}$$

$$\begin{split} S(a, b, z) &= \mathbb{I}_{\{a \leq \underline{a}(\cdot)\}} 0 + \mathbb{I}_{\{a > \underline{a}(\cdot)\}} \left\{ ax - \overline{u} - \kappa \mathbf{v}(\cdot) \right. \\ &- (1 - \tau)b + p(b'(\cdot), z)b'(\cdot)\left[(1 - s) + q(z)\mathbf{v}(\cdot)\right] \\ &+ (1 - s)\beta \mathbb{E}_{a', z'} \left[ S(a', b'(\cdot), z') |z \right] \\ &+ \beta \mathbb{E}_{a', z'} \left[ q(z)\mathbf{v}(\cdot) \underbrace{e(a', b'(\cdot), z')}_{\gamma S(a', b'(\cdot), z')} - f(z) \underbrace{g(a', B(z'), z')}_{(1 - \gamma) S(a', B(z'), z')} |z \right] \right\} \end{split}$$

Return

$$\begin{split} S(a, b, z) &= \mathbb{I}_{\{a \leq \underline{a}(\cdot)\}} 0 + \mathbb{I}_{\{a > \underline{a}(\cdot)\}} \Big\{ ax - \overline{u} - \kappa \mathbf{v}(\cdot) \\ &- (1 - \tau)b + p(b'(\cdot), z)b'(\cdot)\left[(1 - s) + q(z)\mathbf{v}(\cdot)\right] \\ &+ (1 - s)\beta \mathbb{E}_{a', z'} \left[ S(a', b'(\cdot), z') | z \right] \\ &+ \beta \mathbb{E}_{a', z'} \left[ q(z)\mathbf{v}(\cdot)\gamma S(a', b'(\cdot), z') - f(z)(1 - \gamma)S(a', B(z'), z') | z \right] \Big] \end{split}$$

Return

$$\begin{split} S(a, b, z) &= \mathbb{I}_{\{a \leq \underline{a}(\cdot)\}} 0 + \mathbb{I}_{\{a > \underline{a}(\cdot)\}} \Big\{ ax - \overline{u} - \kappa \mathbf{v}(\cdot) \\ &- (1 - \tau)b + p(b'(\cdot), z)b'(\cdot) \left[ (1 - s) + q(z)\mathbf{v}(\cdot) \right] \\ &+ (1 - s)\beta \mathbb{E}_{a', z'} \left[ S(a', b'(\cdot), z') | z \right] \\ &+ \beta \mathbb{E}_{a', z'} \left[ q(z)\mathbf{v}(\cdot)\gamma S(a', b'(\cdot), z') - f(z)(1 - \gamma)S(a', B(z'), z') | z \right] \Big\} \end{split}$$

Return

$$\begin{split} S(a, b, z) &= \max_{d, nd} \left\{ 0, \max_{v, b'} \left\{ ax - \bar{u} - \kappa v \right. \\ &- (1 - \tau)b + p(b', z)b' \left[ (1 - s) + q(z)v \right] \\ &+ (1 - s)\beta \mathbb{E}_{a', z'} \left[ S(a', b', z') | z \right] \\ &+ \beta \mathbb{E}_{a', z'} \left[ q(z)v\gamma S(a', b', z') - f(z)(1 - \gamma)S(a', B(z'), z') | z \right] \right\} \end{split}$$

• Let S(a, b, z) = e(a, b, z) + g(a, b, z) be the joint surplus of a match.

$$\begin{split} S(a, b, z) &= \max_{d, nd} \left\{ 0, \max_{v, b'} \left\{ ax - \bar{u} - \kappa v \right. \\ &- (1 - \tau)b + p(b', z)b' \left[ (1 - s) + q(z)v \right] \\ &+ (1 - s)\beta \mathbb{E}_{a', z'} \left[ S(a', b', z') | z \right] \\ &+ \beta \mathbb{E}_{a', z'} \left[ q(z)v\gamma S(a', b', z') - f(z)(1 - \gamma)S(a', B(z'), z') | z \right] \right\} \end{split}$$

#### Proposition

Firm's policies - b'(b, z), v(b, z) and  $\underline{a}(b, z)$  - maximize the value of a match S(a, b, z).

- **Perturbation**: increase n' by  $\phi$ , keeping debt per worker b' fixed.
- o Need: vacancies  $\Delta \bar{v} = rac{\phi n'}{q(z)}$  and debt  $\Delta \bar{b}' = \phi b'.$

- **Perturbation**: increase n' by  $\phi$ , keeping debt per worker b' fixed.
- Need: vacancies  $\Delta \bar{v} = rac{\phi n'}{q(z)}$  and debt  $\Delta \bar{b}' = \phi b'.$
- **Cost:**  $\Delta \bar{v} = \frac{\kappa \phi n'}{q(z)}$

- **Perturbation**: increase n' by  $\phi$ , keeping debt per worker b' fixed.
- Need: vacancies  $\Delta \bar{v} = rac{\phi n'}{q(z)}$  and debt  $\Delta \bar{b}' = \phi b'.$
- **Cost:**  $\Delta \bar{v} = \frac{\kappa \phi n'}{q(z)}$
- o Benefits:
  - + Surplus:  $\phi n' \gamma \beta \mathbb{E} \left[ S(a', b'(\cdot), z') | z \right]$
  - + New debt:  $p(b'(\cdot), z)\phi \bar{b}'$

- **Perturbation**: increase n' by  $\phi$ , keeping debt per worker b' fixed.
- Need: vacancies  $\Delta \bar{v} = rac{\phi n'}{q(z)}$  and debt  $\Delta \bar{b}' = \phi b'$ .
- Cost:  $\Delta \bar{v} = \frac{\kappa \phi n'}{q(z)}$
- o Benefits:
  - + Surplus:  $\phi n' \gamma \beta \mathbb{E} \left[ S(a', b'(\cdot), z') | z \right]$
  - + New debt:  $p(b'(\cdot), z)\phi \bar{b}'$
- Optimally

$$\kappa \frac{\phi \mathbf{n}'}{q(z)} = p(\mathbf{b}'(\cdot), z)\phi \overline{\mathbf{b}}' + \phi \mathbf{n}' \gamma \beta \mathbb{E} \left[ S(\mathbf{a}', \mathbf{b}'(\cdot), z') | z \right]$$

- **Perturbation**: increase n' by  $\phi$ , keeping debt per worker b' fixed.
- Need: vacancies  $\Delta \bar{v} = \frac{\phi n'}{q(z)}$  and debt  $\Delta \bar{b}' = \phi b'$ .
- **Cost:**  $\Delta \bar{v} = \frac{\kappa \phi n'}{q(z)}$
- Benefits:
  - + Surplus:  $\phi n' \gamma \beta \mathbb{E} \left[ S(a', b'(\cdot), z') | z \right]$
  - + New debt:  $p(b'(\cdot), z)\phi \bar{b}'$
- Optimally

$$\kappa \frac{1}{q(z)} = p(\boldsymbol{b}'(\cdot), z)\boldsymbol{b}' + \gamma \beta \mathbb{E}\left[S(\boldsymbol{a}', \boldsymbol{b}'(\cdot), z') | z\right]$$



o From firm's optimal conditions

$$\boldsymbol{b}'(b,z) = \tau \lambda \frac{\mathbb{E}_{z'} \left[ 1 - H \left( \underline{\boldsymbol{a}}(\boldsymbol{b}'(b,z), z') \right) \right]}{-\partial \boldsymbol{p}(\boldsymbol{b}'(b,z), z) / \partial \boldsymbol{b}'} + \frac{1 - \lambda}{1 - s} \boldsymbol{b}$$

 $\begin{array}{l} + \ \mbox{Low } \lambda \ \mbox{(long maturity), adds persistence to debt!} \\ + \ \mbox{Need } s < \lambda \ \mbox{for a stationary model } ... \end{array}$ 

Return

Parameter	Value	Target/Source
β	0.96 <sup>1</sup> 2	Annual risk-free rate 1%
u	0.5	Standard
$\gamma$	0.5	Standard
$\kappa$	17	Finding $pprox$ 45%
S	0.033	3.5% monthly separation rate
ū	0.6	Shimer (2005) - Hagedorn and Manovskii (2008)
au	13%	U.S. Government Accountability Office
$\lambda$	1/24	2 year debt maturity
$\sigma_{a}$	0.2	Annual default rate 1%
$(\rho_x, \sigma_x)$	(0.98, 0.005)	Standard

Table: Parameter values



Question: How important is labor as an asset to the firm?

- A separation shock
  - + Employment decreases 3%.
  - + One period sock.
  - + Unexpected shock!
- o Compute model response

Return



#### Figure: Model Response to a separation shock

Note: Impulse responses correspond to the average over 50,000 independently simulated economies, all which experienced the same productivity innovation at t = 0.

Julio Blanco and Gaston Navarro "

"Unemployment Accelerator"

# Distance to Default (DD)Computation

#### $\textbf{Theory}(\mathsf{-ish})$

- + Firms total value evolve as:  $dA_t = \mu_A dt + \sigma_A dW_t$
- + "Default point"  $L_t$ , in T periods.
- + Value of equity (Merton ,1970)

$$\begin{aligned} S_t &= A_t \Phi(d_1) - L_t e^{-r_t} \Phi(d_2) \\ d_1 &= \frac{1}{\sigma_A} \left( \ln \left( \frac{A_t}{L_t} \right) + \left( r_t + \frac{1}{2} \sigma_A^2 \right) \right), \qquad d_2 = d_1 - \sigma_A \end{aligned}$$

+ DD: size of shock that induces default (in units of  $\sigma_A$ )

$$DD_t = \frac{\ln\left(\frac{A_t}{L_t}\right) + \left(\mu_A - \frac{1}{2}\sigma_A^2\right)T}{\sigma_A\sqrt{T}}$$

+ Probability of default =  $\Phi(-DD_t)$ 

# Distance to Default (DD)Computation

#### Theory(-ish)

- + Firms total value evolve as:  $dA_t = \mu_A dt + \sigma_A dW_t$
- + "Default point"  $L_t$ , in T periods.
- + Value of equity (Merton ,1970)

$$\begin{aligned} S_t &= A_t \Phi(d_1) - L_t e^{-r_t} \Phi(d_2) \\ d_1 &= \frac{1}{\sigma_A} \left( \ln \left( \frac{A_t}{L_t} \right) + \left( r_t + \frac{1}{2} \sigma_A^2 \right) \right), \qquad d_2 = d_1 - \sigma_A \end{aligned}$$

+ DD: size of shock that induces default (in units of  $\sigma_A$ )

$$DD_t = \frac{\ln\left(\frac{A_t}{L_t}\right) + \left(\mu_A - \frac{1}{2}\sigma_A^2\right)T}{\sigma_A\sqrt{T}}$$

+ Probability of default =  $\Phi(-DD_t)$ 

#### Implementation:

- + Data:  $S_t$  = stock market value,  $L_t$  = short-term debt +  $\frac{1}{2}$  long-term debt.
- + Solve for  $A_t$  as a fixed point problem.
- + Compute  $\delta_t$ .
- + Daily data, averaged to quarterly frequency.

# Distance to Default (DD) Computation



#### Figure: Distance to Default

Note: Distance to Default for non-financial corporate sector, sample period 1975q1 - 2014q4. Computed using CRSP-Compustat merged panel for 15,320 firms.

#### More volatile labor market, more volatile default risk Return





Note: Non-financial corporate sector, period 1975q1 - 2014q4. STD on equation residuals. Data source CRSP-Compustat merged panel for 15,320 firms.

Julio Blanco and Gaston Navarro "Unemployment Accelerator"

#### More volatile labor market, more volatile default risk Return

$$\ln \mathbb{V} \left( \Phi_s \right) = 0.79 + \frac{0.04}{_{[0.03 \ 0.05]}} \ln \mathbb{V} \left( \Delta E_s \right) \qquad R^2 = 0.4\%$$

Prob of default vs Employment growth – STD by sector



Note: Non-financial corporate sector, period 1975q1 - 2014q4. STD on original variables. Data source CRSP-Compustat merged panel for 15,320 firms.

	Julio Blanco and Gaston Navarro	"Unemployment Accelerator"
--	---------------------------------	----------------------------

#### More volatile labor market, more volatile default risk Return



Note: Non-financial corporate sector, period 1975q1 - 2014q4. STD on equation residuals. Data source CRSP-Compustat merged panel for 15,320 firms.

Julio Blanco and Gaston Navarro "Unemployment Accelerator"