

# On the optimal design of transfers and income-tax progressivity\*

Axelle Ferriere  
PSE & CNRS

Philipp Grübener  
Goethe University

Gaston Navarro  
Federal Reserve Board

Oliko Vardishvili  
Yale University

April 4, 2022

## Abstract

We study the optimal design of means-tested transfers and progressive income taxes. In a simple analytical model, we demonstrate an optimally negative relation between transfers and income-tax progressivity due to *efficiency* and *redistribution* concerns. In a rich dynamic model, we quantify the optimal plan with flexible tax-and-transfer functions. Transfers should be larger than currently in the U.S. and financed with moderate income-tax progressivity. Transfers are key to implement higher progressivity in average than in marginal tax-and-transfer rates, achieving redistribution while preserving efficiency. Quantitatively, the left tail of the income distribution determines optimal transfers, whereas the right tail determines income-tax progressivity.

*Keywords:* Heterogeneous Agents, Fiscal Policy, Optimal Taxation, Redistribution.

*JEL:* E21, E62, H21, H23, H53.

---

\*axelle.ferriere@psemail.eu, gruebener@econ.uni-frankfurt.de, gaston.m.navarro@frb.gov, oliko.vardishvili@yale.edu. This work was supported by computational resources provided by the BigTex High Performance Computing Group at the Federal Reserve Bank of Dallas. An earlier version of this paper circulated under the title “Larger transfers financed with more progressive taxes? On the optimal design of taxes and transfers.” We thank Árpád Ábrahám, Isaac Baley, Anmol Bhandari, Moritz Drechsel-Grau, Dirk Krueger, Abdoulaye Ndiaye, Facundo Piguillem, Dominik Sachs, Kjetil Storesletten, Gianluca Violante, and workshop and seminar participants at the IIPF Glasgow 2019, BGSE Summer Forum: Macroeconomics and Social Insurance 2020, EEA 2020, ESWC 2020, German Economic Association 2020, SED Minneapolis 2021, NBER SI Macro Public Finance 2021, CESifo Public Economics Webinar 2021, 52nd Konstanz Seminar, NTA 2021, EUI, Danmarks Nationalbank, FU Berlin, Goethe University, Tinbergen Institute, University of Manchester, University of Chile, Simon Fraser, Collège de France, and Arizona State University for helpful comments and suggestions. The views expressed are those of the authors and not necessarily those of the Federal Reserve Board or the Federal Reserve System.

# 1 Introduction

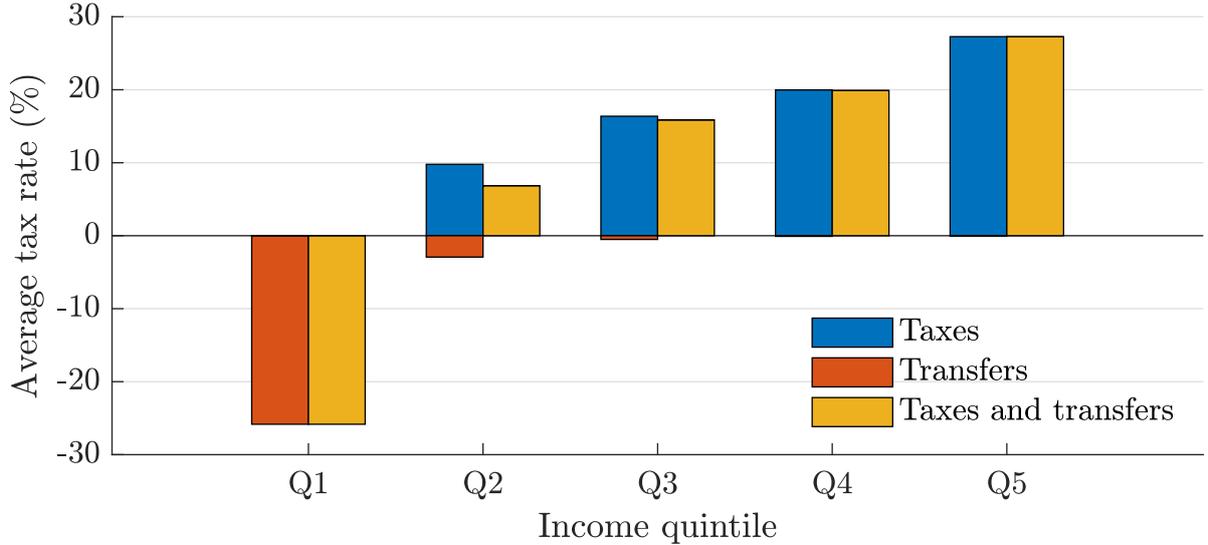
High levels of inequality have made redistributive policies a core topic in recent policy debates. Two key components of redistributive policies are means-tested transfers and progressive income taxes. Both policies can significantly alter the income distribution. In the United States, targeted transfers amount to about 25% of income for the poorest income quintile, while income taxes reduce richest-quintile income by a similar percentage (see Figure 1). The optimal design of these two tools, targeted transfers and progressive income taxes, is thus of paramount importance. A recurring question has been, should transfers be more generous and, if so, should they be financed with more progressive taxes?

In this paper, we study the joint optimal design of targeted transfers and progressive income taxes. We do so in two steps. We first develop a simple analytical model to characterize the optimal relation between transfers and income-tax progressivity. Second, we calibrate a rich dynamic model of the U.S. economy and use it to quantify the findings of the analytical model. We pursue a Ramsey approach and endow a planner with flexible functions for transfers and income taxes, which we refer to as the tax-and-transfer ( $t&T$ ) system.

Our approach has several advantages. The functional forms we use resemble current policies implemented by many countries. Thus, our analysis sheds light on the optimal use of currently available instruments. Furthermore, the Ramsey approach is suitable for a rich quantitative evaluation with empirically realistic efficiency and redistribution concerns. Finally, the instruments we use are simple and characterized by few economically intuitive parameters. Yet, they are flexible enough to generate nonlinear, and potentially non-monotonic, overall  $t&T$  schedules, features often found to be optimal in the Mirrleesian literature. Thus, the flexibility of our Ramsey approach allows us to build intuition on the typical optimal taxation tradeoffs.

We present two main findings. First, we demonstrate an optimally negative relation between transfers and income-tax progressivity—that is, between the intercept of the  $t&T$  system and the progressivity of the marginal  $t&T$  rates. Thus, given economic fundamentals, higher transfers should be financed with lower income-tax progressivity. Second, the optimal  $t&T$  system features more progressive average than marginal rates. This property is implemented with large transfers, to achieve redistribution, and thus low income-tax progressivity, to preserve efficiency. We present these findings using the analytical framework and then evaluate these tradeoffs in the quantitative model. We further use the quantitative model to analyze how the income distribution shapes the optimal  $t&T$  system, and to explore the quantitative importance of non-monotonic marginal  $t&T$  rates on welfare.

We develop an analytical framework to discuss the optimal negative relation between



**Figure 1:** Redistribution in the U.S.

**Notes:** This figure shows average tax and transfer rates by income quintile for the working-age population in 2013. Data is from the Congressional Budget Office (CBO). Details on the components of taxes and transfers are included in Appendix B.1.

transfers and income-tax progressivity. We build on the work in [Heathcote, Storesletten, and Violante \(2017\)](#) by assuming a continuum of households subject to idiosyncratic labor risk and endowing a planner with a log-linear income tax and a lump-sum transfer. Households face a static consumption-labor supply decision. We use local approximations to derive a closed-form formula for welfare that shows the optimal negative relation between transfers and income-tax progressivity.

The welfare formula shows that the optimally negative relation is due to both *efficiency* and *redistribution* concerns. Higher income-tax progressivity and larger transfers both discourage labor supply. In turn, a planner finances larger transfers with lower progressivity to restore labor supply incentives. We refer to this channel as the *efficiency* concern. In terms of *redistribution*, a planner aims to decrease dispersion in consumption. Higher income-tax progressivity and larger transfers both reduce consumption dispersion. The larger the tax progressivity, the lower the consumption dispersion and the smaller the welfare gain from transfers. Thus, larger transfers are optimally financed with less progressive income taxes because of both *efficiency* and *redistribution* concerns. We then investigate the optimal size of transfers in this simple set-up. An illustrative calibration suggests that transfers should be large, and thus income-tax progressivity should be low. Generous transfers with low income-tax progressivity implement more progressive average than marginal  $t&T$  rates, a feature that we robustly find to be welfare improving. This result echoes [Saez \(2001\)](#), which argues that high marginal tax rates at the bottom of the income distribution are optimal to finance a lump-sum.

We confirm these analytical findings in a rich, quantitative Bewley-Huggett-Aiyagari

incomplete-market model calibrated to the U.S. economy. We enrich the set of fiscal instruments and endow the planner with targeted transfers that, in line with current practices, phase out with both labor and capital income. The phase-out of transfers, combined with progressive income taxes, allows for non-monotonic marginal  $t&T$  rates. We also incorporate an empirically realistic process for idiosyncratic labor risk, with a thick right tail of productivity and higher-order moments of income risk, to accurately capture redistribution needs in the economy. The model produces labor supply elasticities and wealth effects in line with the data and thus accurately captures efficiency concerns as well. We consider once-and-for-all changes to the fiscal instruments and incorporate transitions toward the new steady state in our welfare computations.

The optimal  $t&T$  system in the quantitative model is substantially more redistributive than the current system in the United States. Optimal transfers amount to \$29,100 (2013 U.S. dollars) per year for the lowest-income household.<sup>1</sup> This number implies an income floor of 45% of median income. Transfers optimally phase out, albeit at a slow rate, such that a household with median income receives a transfer of about \$8,200. In line with our analytical findings, large transfers are optimally financed with moderate income-tax progressivity, at a level similar to the current U.S. system. The optimal system implies large welfare gains of 9.64% in consumption equivalent terms. These welfare gains are largely due to better insurance and redistribution. In addition, despite a large drop in aggregate labor, there are small efficiency improvements because of a better allocation of hours worked across households. While welfare gains primarily accrue to the poor, 79% of households benefit from the reform.

A substantial part of the welfare gains in the benchmark plan can be achieved with either of two common tax proposals: a universal basic income (UBI) or an affine plan. In terms of tax instruments, the UBI plan eliminates the phase-out of transfers but still optimizes the progressivity of income taxes. The affine plan is a UBI financed with flat income taxes. The optimal UBI plan implies a lump-sum transfer of \$28,800 per household, financed with barely progressive income taxes: the income-tax rate averages 60% for the poorest income quintile and 64% for the richest income quintile. As the optimal UBI plan features almost flat taxes, the optimal affine plan is similar to the UBI one. The affine plan features a lump-sum transfer of \$30,000 and a tax rate of 64%. The welfare gain of the UBI is 9.43% in consumption equivalent terms, while the affine gains are 9.38%, both close to the gains in the benchmark plan. Thus, our framework is supportive of a lump-sum transfer, provided that it is financed with the right income-tax progressivity. Alternatively, if implementing income-tax rates above 60% is infeasible, a planner should favor a phase-out of transfers, as it is associated with lower tax rates.

The UBI and affine experiments point to the importance of disentangling average

---

<sup>1</sup>As we discuss in Section 3.3, we report transfers per household, as it is the unit of calibration of the quantitative model.

and marginal rates and shed light on the relative importance of the two standard elements in the Mirrlees literature, the positive intercept and the U-shape of the marginal rates. Transfers, even if lump-sum, allow to separate the progressivity of average and marginal  $t&T$  rates. In order to redistribute while preserving efficiency, the optimal plan robustly features more progressive average than marginal rates. Additionally, the phasing-out of transfers allows for non-monotonic marginal rates. Welfare gains come primarily from disentangling the progressivity of average and marginal  $t&T$  rates, whereas the phase-out of transfers allows for lower income-tax rates.

Finally, we use our quantitative model to show that the income distribution shapes the negative relation between transfers and income-tax progressivity. In particular, the right tail of the income distribution shifts this relation: the lower the income concentration at the top, the lower the income-tax progressivity for each level of transfer. Yet, income concentration at the top barely affects the optimal level of transfers, and only the income-tax progressivity adjusts. In contrast, when bottom-income households are richer, the optimal income-tax progressivity for each transfer does not change, but the optimal transfers decrease. Thus, optimal transfers are driven by the left tail of the income distribution, whereas income-tax progressivity is determined by the right tail of the distribution.

**Related literature.** This paper belongs to a large literature on optimal taxation. Seminal papers in the Mirrleesian tradition include [Mirrlees \(1971\)](#), [Diamond \(1998\)](#), and [Saez \(2001\)](#). At a general level, our Ramsey approach with flexible  $t&T$  functional forms allows for a quantitative evaluation of the main tradeoffs discussed in this literature within a rich dynamic model.

Our paper relates more closely to [Heathcote and Tsujiyama \(2021\)](#), which compares a fully nonlinear Mirrleesian plan to a log-linear and an affine tax system in a static environment. While not solving for the Mirrleesian plan, we analyze a more flexible Ramsey plan in a dynamic general equilibrium model. As we explain in [Section 2](#), the negative relation we derive between transfers and income-tax progressivity is reminiscent of, yet different from, the negative relation [Heathcote and Tsujiyama \(2021\)](#) derive between spending and income-tax progressivity. In contrast to them, we find that allowing for transfers generates larger welfare gains than allowing for increasing marginal rates—see [Section 4](#). Finally, they also find that income concentration at the top increases optimal tax progressivity. We further explore how the tails of the income distribution alter the optimal relation between transfers and income-tax progressivity. Also related to our paper are [Chang and Park \(2021\)](#) and [Park \(2022\)](#), which use a variational approach to derive a tax formula in an Aiyagari model with [Greenwood, Hercowitz, and Huffman \(1988\)](#) preferences. Our Ramsey approach allows for intuitive insights, as well as the

inclusion of transitions from the calibrated steady-state in welfare computations.<sup>2</sup>

Our quantitative framework relates to several papers interested in optimal tax progressivity in incomplete-market models. An early contribution is [Conesa and Krueger \(2006\)](#), and subsequent work has focused on: transitional dynamics ([Bakiş, Kaymak, and Poschke 2015](#)), superstars and entrepreneurs ([Brüggemann 2021](#); [Kindermann and Krueger 2021](#)), human capital accumulation ([Badel, Huggett, and Luo 2020](#); [Krueger and Ludwig 2016](#); [Peterman 2016](#)), negative income tax programs ([Lopez-Daneri 2016](#)), and Laffer curves ([Guner, Lopez-Daneri, and Ventura 2016](#); [Holter, Krueger, and Stepanchuk 2019](#)). Our paper also relates to recent quantitative analyses of universal basic income policies, with a specific focus on human capital ([Daruić and Fernández 2021](#); [Luduvic 2021](#)), labor market frictions ([Jaimovich, Saporta-Eksten, Setty, and Yedid-Levi 2022](#); [Rauh and Santos 2022](#)), and consumption taxes ([Conesa, Li, and Li 2021](#)).

Two recent works analyze transfers and progressive taxes in a rich quantitative setup. [Boar and Midrigan \(2021\)](#) focuses more specifically on the role of wealth taxes in a framework with entrepreneurs. [Guner, Kaygusuz, and Ventura \(2021\)](#) develops a rich modeling of the household and a detailed calibration of the current U.S.  $t&T$  system. In line with our quantitative finding, both papers find that an affine plan is welfare improving. Our Ramsey approach with flexible tax instruments allows for new insights on the tradeoff between transfers and income-tax progressivity. Additionally, the phase-out of transfers allows us to consider non-monotonic marginal rates, a common property of the optimal Mirrleesian  $t&T$  system.

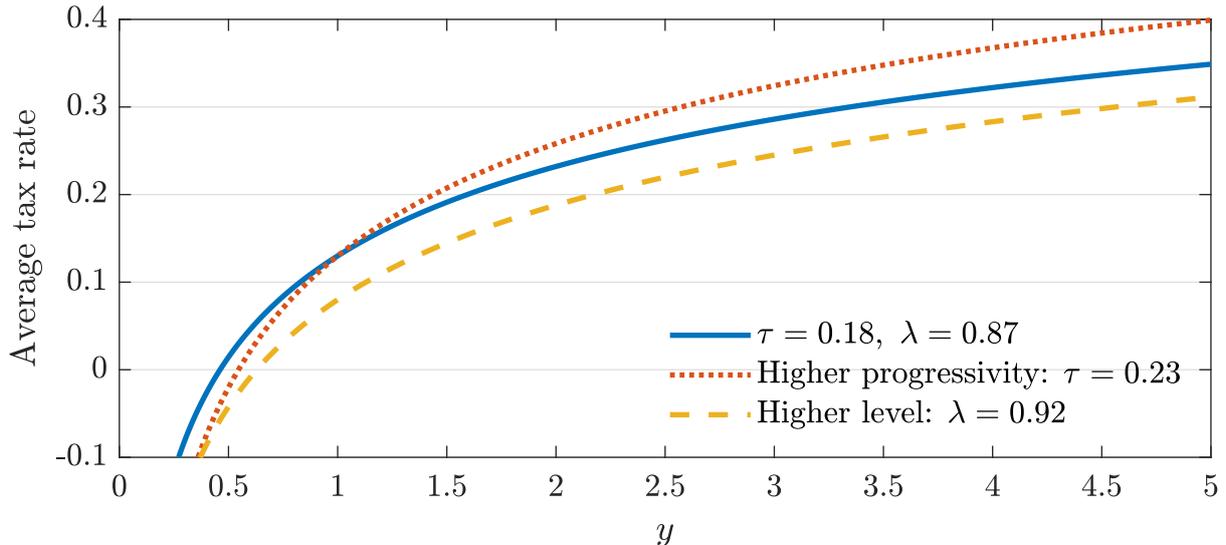
**Roadmap.** Section 2 presents the analytical model. Section 3 contains the quantitative model and its calibration. Section 4 analyzes the optimal fiscal plan. Section 5 discusses the effect of income distribution on optimal transfers and income-tax progressivity. Section 6 concludes.

## 2 An analytical model

We start with a simple analytical model to show the optimal relationship between the size of transfers and the income-tax progressivity. We build on [Heathcote, Storesletten, and Violante \(2017\)](#) and propose a tractable heterogeneous-agent model in which a government uses progressive income taxes to finance public spending and a lump-sum transfer. We first analyze the relationship between transfers and income-tax progressivity in the representative-agent case. Next, we consider heterogeneity. We use local approximations to analytically show that the welfare gains of transfers decline with income-tax progressivity, and resort to numerical methods to verify the accuracy of the approxima-

---

<sup>2</sup>Our paper also relates to advances in New Dynamic Public Finance such as [Kapićka \(2013\)](#), [Farhi and Werning \(2013\)](#), [Golosov, Troshkin, and Tsyvinski \(2016\)](#), [Findeisen and Sachs \(2017\)](#), [Stantcheva \(2017\)](#) and [Boerma and McGrattan \(2020\)](#).



**Figure 2:** Log-linear tax function

**Notes:** This figure shows the average tax rate for various parameterizations of the log-linear tax function.

tions. Our numerical findings also suggest that optimal transfers should be large, a result that we confirm in the quantitative model with endogenous self-insurance in Section 3.

## 2.1 Environment: a static Bewley-Hugett economy

The economy is populated by a continuum of ex-ante homogenous households, a representative firm, and a utilitarian government. Households are hand-to-mouth, value consumption  $c$  and leisure  $1 - n$ , and their labor productivity  $z$  follows a stochastic Markov process. The representative firm uses a linear technology to transform labor into output. The government finances exogenous government spending  $G$  and a lump-sum transfer  $T$  with log-linear labor taxes.<sup>3</sup>

*Taxes.*—A household with labor income  $y$  pays taxes  $\mathcal{T}(y) = y - \lambda y^{1-\tau}$ , where  $\tau$  captures the progressivity and  $\lambda$  the level of taxes. For  $\tau = 0$ , tax rates are flat and equal to  $1 - \lambda$ . When  $\tau > 0$  ( $\tau < 0$ ), marginal and average tax rates are increasing (decreasing) in income. Figure 2 shows the tax function.

*Households.*—In period  $t$ , household  $i$  chooses consumption  $c_{it}$  and labor  $n_{it}$  to maximize utility

$$u(c_{it}, n_{it}) = \ln c_{it} - B \frac{n_{it}^{1+\varphi}}{1+\varphi} \quad (1)$$

<sup>3</sup>This tax function has been widely used since Feldstein (1969), and more recently by Benabou (2000) and Heathcote, Storesletten, and Violante (2017).

subject to a static budget constraint

$$c_{it} = \lambda (z_{it}n_{it})^{1-\tau} + T. \quad (2)$$

We assume a log-AR(1) process for  $z$ , with  $v_\omega$  controlling the degree of heterogeneity across households

$$\log z_{it} = \rho \log z_{it-1} + \omega_{it}, \quad \omega_{it} \sim \mathcal{N}\left(-\frac{v_\omega}{2(1+\rho)}, v_\omega\right).$$

Note that log utility, together with hand-to-mouth households, ensures a closed-form solution for labor when transfers are zero, a property that will be useful in deriving the welfare formula.<sup>4</sup>

*Government.*— The government budget constraint is

$$G + T = \int z_{it}n_{it}di - \lambda \int (z_{it}n_{it})^{1-\tau} di. \quad (3)$$

*Technology.*— The resource constraint is

$$\int c_{it}di + G = \int z_{it}n_{it}di. \quad (4)$$

## 2.2 Representative agent

We first abstract from heterogeneity and assume  $v_\omega = 0$ .

*First-best.*—Let  $n^*(G)$  denote the first-best allocation, which maximizes the utility of the representative agent given the resource constraint (4). It is characterized by

$$Bn^\varphi(n - G) = 1. \quad (5)$$

*Second-best.*—We turn to the optimal allocation when spending is financed with a progressive income tax and a transfer.

**Claim 1** *For all levels of transfers  $T < n^*(G) - G$ , the efficient allocation  $n^*(G)$  can be implemented with progressivity  $\tau^*(G, T)$  equal to*

$$\tau^*(G, T) = -\frac{G + T}{n^*(G) - (G + T)}. \quad (6)$$

---

<sup>4</sup>In the absence of transfers, a no-trade theorem applies with log utility when  $z$  follows a random walk, as shown in [Heathcote, Storesletten, and Violante \(2017\)](#). We deviate from the no-trade conditions by including transfers. We further deviate by assuming a log-AR(1) process for  $z$ , which simplifies the analytical results. The no-saving assumption is relaxed in the quantitative model of Section 3. We have also derived an analytical relation between optimal transfers and progressivity using the structure of stochastic shocks in [Heathcote, Storesletten, and Violante \(2017\)](#). While the expression gets cumbersome, the negative relationship between progressivity and transfers remains.

As such, the optimal income tax progressivity  $\tau$  decreases with transfers  $T$ .

All proofs in this section are relegated to Appendix A.

To understand why the first-best allocation can always be implemented regardless of the level of transfers, we first focus on the case when  $T = 0$ . Maximizing the household's utility (1) given the budget constraint (2) yields an optimal labor policy function in closed form,

$$n = \left( \frac{1 - \tau}{B} \right)^{\frac{1}{1+\varphi}} \equiv n_0(\tau), \quad (7)$$

where the 0 subscript stands for zero transfers. Equation (7) shows a one-to-one negative relationship between  $n$  and  $\tau$ , which ensures that a planner can always pick the progressivity  $\tau$  to implement  $n^*(G)$ . Thus, as the first-best allocation  $n^*(G)$  increases with  $G$ , the optimal  $\tau$  declines with  $G$ . This negative relation between  $G$  and  $\tau$  relates to the notion of “fiscal pressure” discussed by [Heathcote, Storesletten, and Violante \(2017\)](#) and further developed by [Heathcote and Tsujiyama \(2021\)](#): a planner finances larger spending with less progressive taxes to incentivize labor supply.<sup>5</sup>

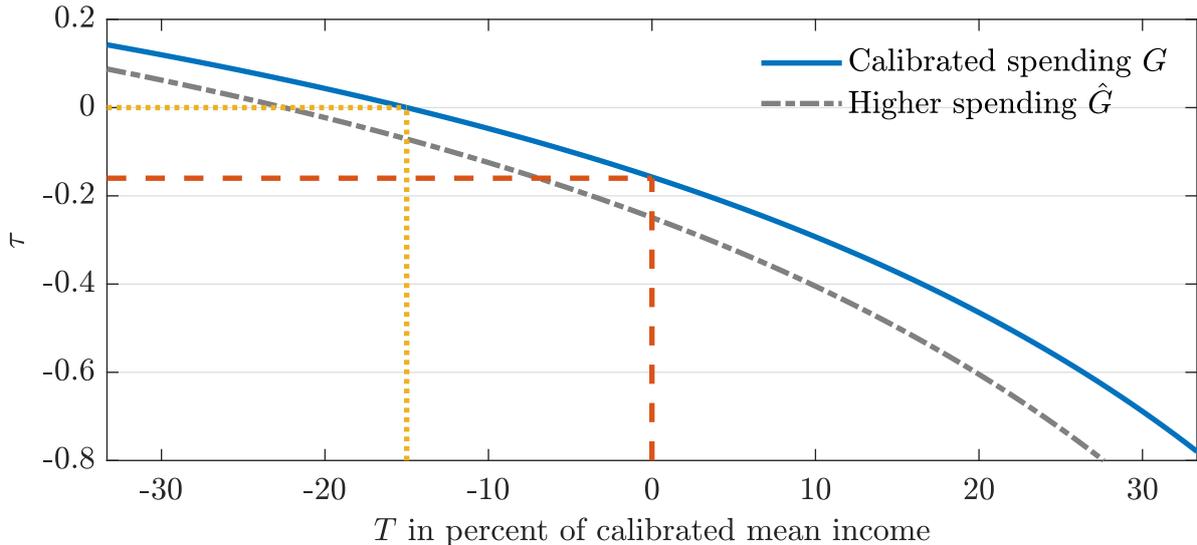
When transfers are not zero, the policy function for labor does not admit a closed-form solution. However, it is easy to show that labor supply decreases with transfers  $T$  because of a wealth effect. This property explains the optimal negative relationship between  $\tau$  and  $T$  in Claim 1. Progressivity falls as transfers increase, such that labor supply remains at  $n^*(G)$  despite the more generous transfers.<sup>6</sup> In this representative-agent set-up, the negative relation between transfers and progressivity is entirely driven by *efficiency* concerns.

Note that the effect of transfers on progressivity is qualitatively different from the one of spending. Larger spending increases the first-best labor supply, but only taxes directly impact the household labor policy. With higher spending, optimal progressivity decreases so that labor supply reaches its new first-best level. Transfers, by contrast, do not alter the first-best labor supply but weaken the household's incentives to provide labor. With higher transfers, optimal progressivity decreases so that labor supply remains at its unchanged first-best level. In short, the efficiency concerns governing the negative relation between  $T$  and  $\tau$  differ from those governing the negative relation between  $G$  and  $\tau$ .

Figure 3 presents a numerical illustration of the relationship between transfers and progressivity for the representative agent case. The Frisch elasticity  $\varphi^{-1}$  is set to 0.4, and the labor disutility parameter  $B$  is chosen such that labor supply  $n_0(\tau) = 0.3$ . Pro-

<sup>5</sup>See [Ayaz, Fricke, Fuest, and Sachs \(2021\)](#) for a related discussion.

<sup>6</sup>Equation (6) retrieves an old result in the Ramsey literature. When  $\tau = 0$ , the optimal transfer is  $T = -G$ . That is, the government finances all spending with a lump-sum tax and sets the distortionary flat tax rate at zero.



**Figure 3:** Representative agent: implementing the first-best allocation

**Notes:** This figure shows the combinations of lump-sum transfer  $T$  and income-tax progressivity  $\tau$  that implement the first-best allocation in the representative-agent case. The dashed line highlights the case of zero transfer, in which progressivity is negative:  $\tau = -0.16$ . The dotted line marks the scenario of an affine tax system,  $\tau = 0$ , in which a lump-sum tax finances all government spending:  $T = -G$ . The dash-dotted line shows the optimal combinations of  $T$  and  $\tau$  for larger spending, at  $\hat{G} = 1.5G$ .

gressivity is fixed at  $\tau = 0.18$ , based on the U.S. estimates of [Heathcote, Storesletten, and Violante \(2017\)](#), and government spending is set to 15% of output. As can be seen, optimal progressivity  $\tau$  declines as transfers  $T$  increase. We also plot the optimal relation between  $\tau$  and  $T$  for higher spending: a larger  $G$  generates a new curve, associated with a new level of labor, while a larger  $T$  moves the optimum on the existing curve.

## 2.3 Heterogeneity

We next consider the economy with heterogeneous agents. In this case, the second-best allocation is not efficient and so cannot be characterized by the first-best allocation. We thus proceed as follows. First, we derive a formula for welfare as a function of progressivity  $\tau$  in the tractable case of  $T = 0$ .<sup>7</sup> Then, we use local approximations around that point to characterize welfare as a function of  $\tau$  and  $T$ .

### 2.3.1 Heterogeneity and no transfers

When transfers are zero, the optimal labor policy is constant across households and, again, equal to  $n_0(\tau) \forall z$ . Given this simple policy function, we can compute output,  $Y_0(\tau) = n_0(\tau)$ , and the tax function level parameter  $\lambda_0(\tau)$  from the government budget

<sup>7</sup>This formula is a special case of [Heathcote, Storesletten, and Violante \(2017\)](#) that, beyond the alternative structure of stochastic shocks, also considers skill investment and preference heterogeneity.

constraint. Then, we can derive aggregate welfare as a function of progressivity  $\tau$  as

$$W(\tau) = \underbrace{\log(n_0(\tau) - G)}_{\text{Size}} \underbrace{-\frac{1-\tau}{1+\varphi}}_{\text{Labor disutility}} \underbrace{-(1-\tau)^2 \frac{v_\omega}{2(1-\rho^2)}}_{\text{Redistribution}}. \quad (8)$$

Efficiency

This expression has a straightforward economic interpretation. The first two terms capture the *efficiency* concerns governing the optimal choice of  $\tau$ . Because progressivity depresses labor supply, a larger  $\tau$  reduces the *size* of the economy, and thus aggregate consumption  $C_0(\tau) = n_0(\tau) - G$ . Yet, a larger  $\tau$  also reduces *labor disutility*  $-B \frac{n^{1+\varphi}}{1+\varphi}$ , which, using equation (7), equals the second term in the formula. The last term is proportional to the variance of log consumption and captures *redistribution* concerns. Larger progressivity reduces dispersion in consumption, which is welfare improving. As such, heterogeneity increases optimal progressivity. Notice that, when  $v_\omega = 0$ , the progressivity  $\tau$  that maximizes (8) implements the first-best allocation, as in Section 2.2.

Figure 4 illustrates the forces determining optimal progressivity  $\tau$ . We calibrate  $\rho = 0.935$  and set  $v_\omega = 0.034$  to match a variance of log consumption of 0.18, as also measured in Heathcote, Storesletten, and Violante (2017). Optimal progressivity is zero with a representative agent and no public spending. Positive spending decreases  $\tau$  to  $-0.16$ , reflecting the spending externality discussed earlier. Adding heterogeneity increases  $\tau$  to 0.29 because of redistribution concerns.

As  $\tau$  is positive, average and marginal tax rates both increase with income. The presence of transfers will loosen the tight link between average and marginal rates, as we discuss next.

### 2.3.2 Heterogeneity and transfers

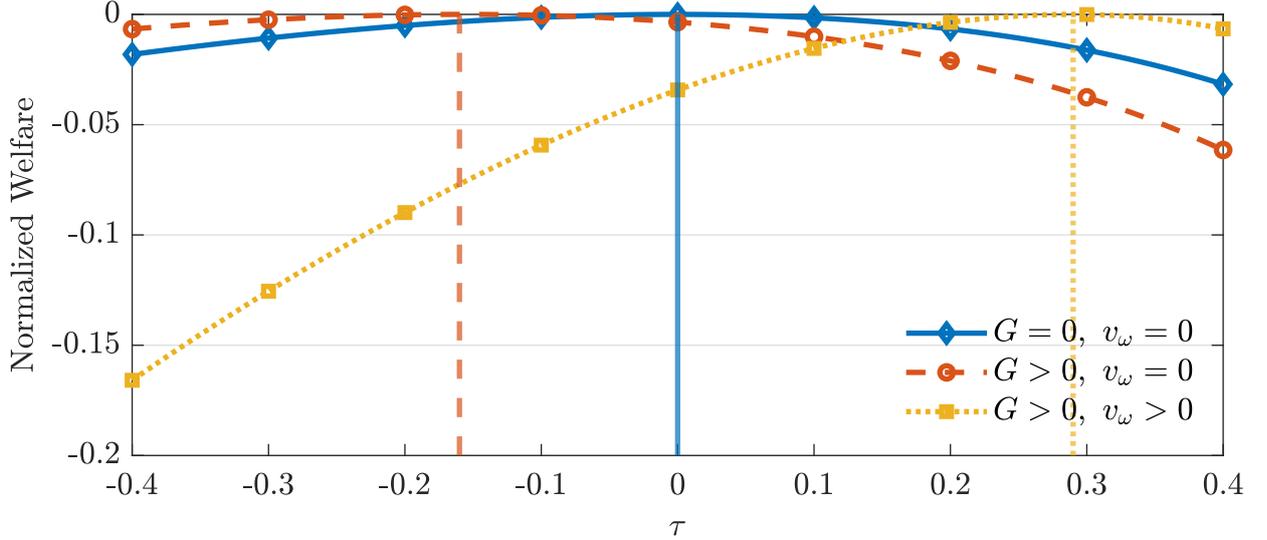
With non-zero transfers, the policy function for labor does not admit a closed-form solution. Instead, we use the implicit function theorem and approximate the policy around the case with  $T = 0$ :

$$\hat{n}_{it} \approx n_0(\tau) - \frac{T}{1+\varphi} \frac{n_0(\tau)}{n_0(\tau) - G} \exp\left(-\tau(1-\tau) \frac{v_\omega}{2(1-\rho^2)}\right) z_{it}^{-(1-\tau)}. \quad (9)$$

Note that labor supply falls in transfers because of the wealth effect.

Using this approximation, we follow similar steps as in the case without transfers to obtain an expression for welfare as a function of progressivity  $\tau$  and transfer  $T$ :

$$W(\tau, T) = W(\tau, 0) + T \left[ \Omega_e^{ra}(\tau) + \Omega_r^{ha}(\tau, v_\omega) + \Omega_e^{ha}(\tau, v_\omega) \right]. \quad (10)$$



**Figure 4:** Optimal income-tax progressivity without transfers

**Notes:** This figure shows welfare as a function of income-tax progressivity when transfers are zero for three cases. Case 1 features no heterogeneity and no exogenous spending requirement. Case 2 features positive exogenous spending, while in case 3 there is also household heterogeneity. Welfare is normalized to zero at the respective optimal progressivity.

The parenthesis captures the marginal effect of transfers  $T$  on welfare for a given progressivity level  $\tau$ . As we show below, the marginal effect of transfers decreases with  $\tau$ , which explains an optimally negative relation between  $\tau$  and  $T$ . This effect can be decomposed in three terms: an *efficiency* term  $\Omega_e^{ra}(\tau)$ , which summarizes the tradeoffs for a representative agent, and two additional terms coming from the cross section of heterogeneous agents: an *efficiency* term  $\Omega_e^{ha}(\tau, v_\omega)$  and a *redistribution* term  $\Omega_r(\tau, v_\omega)$ . We discuss each term next.

The first term is independent of heterogeneity  $v_\omega$  and captures the *efficiency* tradeoff described in the representative agent case:

$$\begin{aligned}
 \Omega_e^{ra}(\tau) &\equiv \underbrace{-\frac{n_0(\tau)}{n_0(\tau) - G} \frac{1}{1 + \varphi}}_{\text{Size}} \underbrace{\frac{1}{n_0(\tau) - G} + \frac{1 - \tau}{1 + \varphi}}_{\text{Labor disutility}} \frac{1}{n_0(\tau) - G} & (10.a) \\
 &= u_c(C_0(\tau)) \left. \frac{\partial Y^{ra}(\tau, T)}{\partial T} \right|_{T=0} + u_n(n_0(\tau)) \left. \frac{\partial n^{ra}(\tau, T)}{\partial T} \right|_{T=0}
 \end{aligned}$$

where  $Y^{ra}$  and  $n^{ra}$  denote output and labor in the representative-agent economy. Output decreases with transfers, and the welfare cost of smaller output is evaluated using the marginal utility of consumption. At the same time, lower hours induce a welfare gain, evaluated using the marginal utility of leisure.

To see how efficiency gains of transfers change with  $\tau$ , we can rearrange  $\Omega_e^{ra}(\tau)$  as

$$\Omega_e^{ra}(\tau) = \frac{1}{1 + \varphi} \frac{1}{n_0(\tau) - G} \left[ \frac{-G}{n_0(\tau) - G} - \tau \right].$$

**Claim 2**  $\partial\Omega_e^{ra}(\tau)/\partial\tau < 0 \forall\tau$ , and  $\Omega_e^{ra}(\tau^*(G, 0)) = 0$ .

The intuition behind  $\Omega_e^{ra}(\tau)$  is analogous to the one in Section 2.2. Overall, the efficiency gains of using a transfer decrease with progressivity  $\tau$ , in line with Claim 1. When  $\tau = \tau^*(G, 0)$ , the first-best is implemented with zero transfers, and there are no efficiency gains from deviating from  $T = 0$ . Hence,  $\Omega_e^{ra}(\tau^*(G, 0)) = 0$ . When  $\tau < \tau^*(G, 0)$ , labor supply is above optimal, so it is welfare improving to use transfers to decrease labor supply and  $\Omega_e^{ra}(\tau) > 0$ . Analogously, labor is below optimal when  $\tau > \tau^*(G, 0)$ , so negative transfers increase welfare and  $\Omega_e^{ra}(\tau) < 0$ .

The second term captures the distribution of wealth effects on the labor supply of heterogeneous agents:

$$\begin{aligned} \Omega_e^{ha}(\tau, v_\omega) &= \frac{\tau(1 - \tau)}{n_0(\tau) - G} \frac{v_\omega}{1 - \rho^2} \frac{n_0(\tau)}{n_0(\tau) - G} \frac{1}{1 + \varphi} \\ &= u_c(C_0(\tau)) \left[ \frac{\partial Y(\tau, T)}{\partial T} \Big|_{T=0} - \frac{\partial Y^{ra}(\tau, T)}{\partial T} \Big|_{T=0} \right]. \end{aligned} \quad (10.b)$$

This term changes non-monotonically with  $\tau$  when  $v_\omega > 0$ . Transfers reduce labor supply more in the presence of heterogeneity, as the larger wealth effect of the poor more than offsets the smaller wealth effect of the rich. Higher progressivity reduces this dispersion of wealth effects, bringing labor supply closer to the representative agent case. However, the effect on output depends not only on labor supply but also on the distribution of  $z$ , which makes the effect of  $\tau$  non-monotonic. In our calibrations,  $\Omega_e^{ha}(\tau, v_\omega)$  is typically small, and  $\partial[\Omega_e^{ra}(\tau) + \Omega_e^{ha}(\tau, v_\omega)]/\partial\tau < 0$  holds quantitatively so that the total efficiency gains of transfers decrease with progressivity.

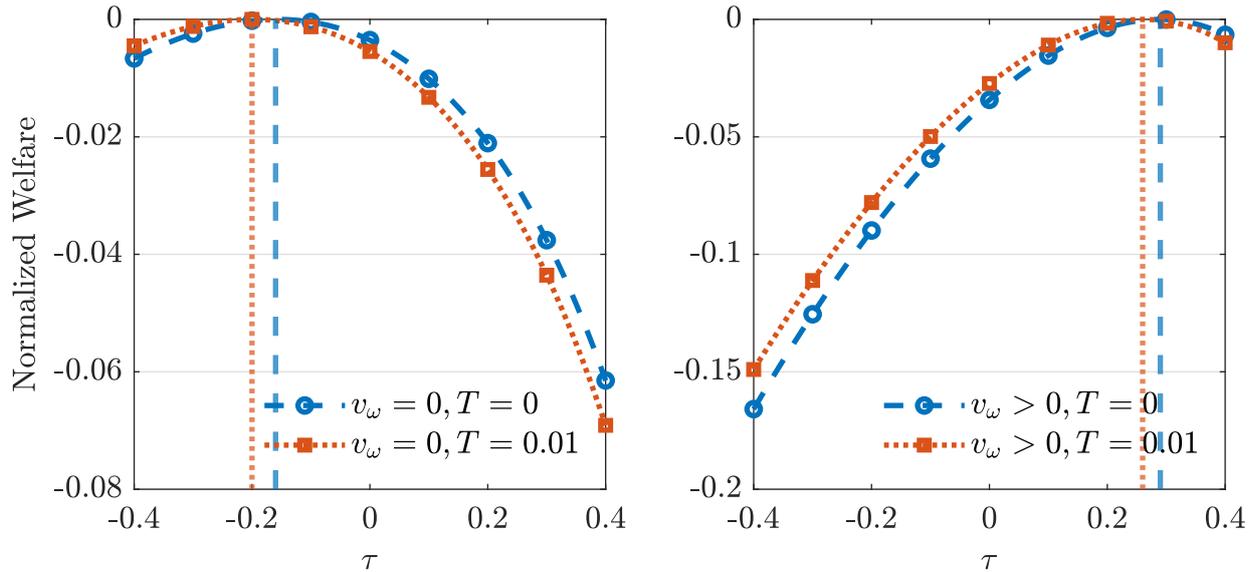
Finally, the last term reflects *redistribution* concerns associated with transfers when agents are heterogeneous:

$$\Omega_r(\tau, v_\omega) = \frac{(1 - \tau)^2}{n_0(\tau) - G} \frac{v_\omega}{1 - \rho^2} = \mathbb{E}[u_c(c_0(\tau))] - u_c(C_0(\tau)), \quad (10.c)$$

where  $c_0(z, \tau)$  is individual consumption as in equation (2).

**Claim 3**  $\partial\Omega_r(\tau, v_\omega)/\partial\tau \leq 0 \forall\tau \in [-1, \hat{\tau}(G)]$ , with  $\hat{\tau}(G) = 1$  for  $G = 0$ .

This term captures dispersion in consumption and is strictly positive as long as  $v_\omega > 0$  and  $\tau < 1$ . Indeed, transfers are welfare improving as they reduce dispersion in marginal



**Figure 5:** Optimal income-tax progressivity with transfers

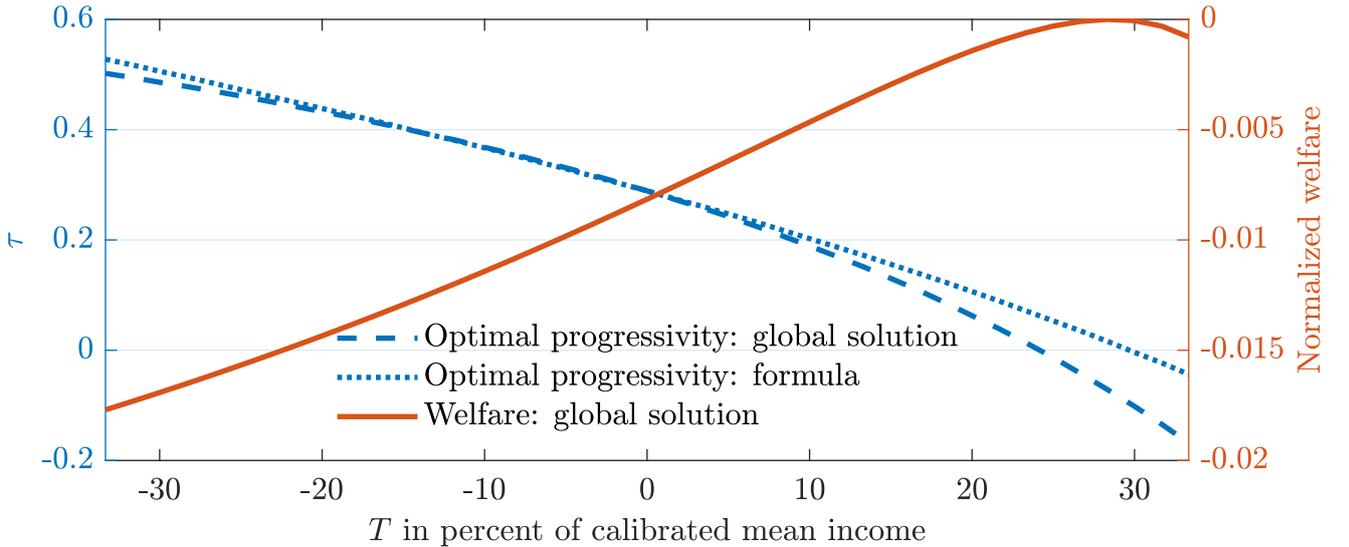
**Notes:** This figure shows welfare as a function of income-tax progressivity, comparing the case of zero transfers and positive transfers for both the representative-agent (left panel) and heterogeneous-agent (right panel) scenarios. Exogenous spending is positive. Welfare is normalized to zero at the respective optimal progressivity.

utilities of consumption. However, under some mild conditions, the redistribution gain of using a lump-sum transfer decreases with progressivity; that is,  $\partial\Omega_r(\tau, v_\omega)/\partial\tau \leq 0$ .<sup>8</sup> This result is intuitive. Larger progressivity already reduces the dispersion in marginal utilities, such that there are fewer gains from reducing consumption dispersion even further with higher transfers. Thus, transfers and progressivity act as substitutes to redistribute resources. In the extreme case of  $\tau = 1$ , after-tax incomes are equalized, and the redistributive gains from having a positive transfer are zero; that is,  $\Omega_r(1, v_\omega) = 0$ . Overall, *redistribution* concerns strengthen the negative optimal relationship between transfers and tax progressivity driven by *efficiency* concerns: the more generous the transfer  $T$ , the smaller the progressivity  $\tau$ .

These *redistribution* concerns further differentiate the effect of transfers on income-tax progressivity from the one of spending. Since higher spending does not alter consumption dispersion, the relationship between  $G$  and  $\tau$  is not altered by redistribution motives.

Figure 5 illustrates how transfers affect optimal progressivity using our approximation around  $T = 0$ . Without heterogeneity, spending is optimally financed with negative progressivity when transfers are zero. Optimal progressivity further declines when transfers are positive, solely for *efficiency* reasons. With heterogeneity, positive transfers again reduce the optimal progressivity, because of both *efficiency* and *redistribution* concerns.

<sup>8</sup>A formal derivation of the bound  $\hat{\tau}(G)$  can be found in Appendix A. The bound is typically close to 1, and exactly 1 when  $G=0$ . In our calibration, it is equal to 0.998.



**Figure 6:** Optimal income-tax progressivity given transfers

**Notes:** This figure compares, for each transfer, the optimal income-tax progressivity implied by the formula to the global (numerical) solution. It also shows, for each transfer, the welfare associated with the optimal progressivity.

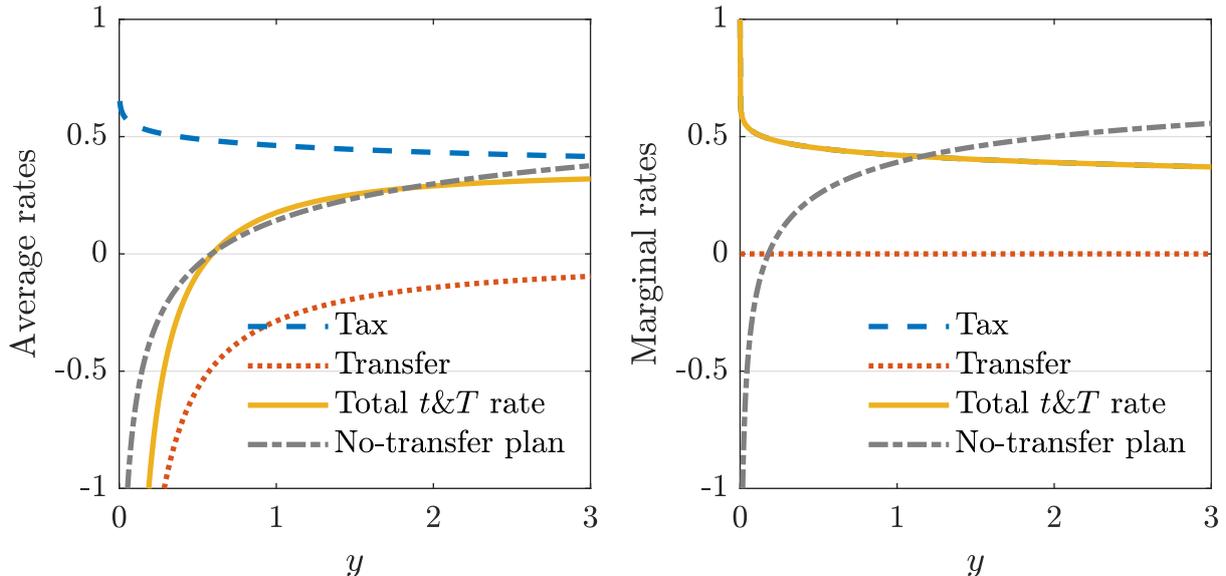
### 2.3.3 Optimal transfers

The welfare formula we derived is insightful on the optimal relationship between  $T$  and  $\tau$ , but it is based on an approximation around  $T = 0$ . Hence, we now pursue a numerical solution of the model. The goal is twofold: to check the accuracy of the approximation and to compute the optimal level of transfers.

The numerical solution confirms the optimal negative relation between transfers  $T$  and progressivity  $\tau$  around and away from  $T = 0$ , as Figure 6 shows. In addition, our approximation accurately characterizes the optimal tax progressivity for a wide range of transfers. Figure 6 also plots welfare as a function of transfers. The highest welfare is achieved with a large lump-sum transfer combined with negative progressivity:  $T$  amounts to roughly 29% of mean calibrated income and income taxes are regressive with  $\tau = -0.076$ .

The optimal  $t&T$  system features progressivity in average rates due to the optimal large transfers. However, the  $t&T$  system is regressive in terms of marginal rates because of the optimal negative  $\tau$ . Figure 7 plots the average and marginal  $t&T$  rates implied by the welfare maximizing policy. Larger progressivity in average than in marginal rates is reminiscent of typical results in the Mirrleesian literature.

Figure 7 also includes the average and marginal  $t&T$  rates of the optimal log-linear plan when transfers are constrained to be zero. In this case, progressivity is high, at  $\tau = 0.29$ , because a positive  $\tau$  is the only tool available for the planner to redistribute. However, in the absence of transfers, redistribution through increasing average rates implies increasing marginal rates. A lump-sum transfer allows to break the tight



**Figure 7:** Optimal tax-and-transfer system: tax rates

**Notes:** This figure shows average (left panel) and marginal rates (right panel) of the optimal tax-and-transfer system. For comparison, the figure also plots optimal average and marginal rates in the no-transfer case.

link between average and marginal rates imposed by the log-linear function, which is welfare improving, as this simple calibration suggests.

### 2.3.4 Revisiting fiscal pressure with optimal transfers

The negative relation between  $G$  and  $\tau$  is flattened when transfers are chosen optimally. Starting from the optimal fiscal system, an increase of  $G$  by 0.01 optimally decreases  $\tau$  to  $-0.124$  when transfers are constrained to be constant. In contrast, if  $T$  optimally adjusts as well,  $T/Y$  decreases to 27%, while  $\tau$  decreases only to  $-0.085$ . Thus, the effect of fiscal pressure on progressivity is attenuated with transfers.

## 2.4 Taking stock

There are two main takeaways from the analytical part. First, due to both *efficiency* and *redistribution* concerns, there is an optimally negative relationship between the size of transfers and the income-tax progressivity. That is, a larger intercept of the  $t&T$  system is optimally associated with less progressive marginal  $t&T$  rates. As such, transfers allow the fiscal plan to disentangle average from marginal rates. Second, the optimal plan features more progressive average than marginal rates. In fact, in this simple model, the optimal transfer is large and income taxes are regressive.

While calibrated, the analytical model is too simple to make a truly quantitative statement about the optimal combination of the instruments. To address this question, we move to a more quantitative macro model of the U.S. economy next.

### 3 A quantitative model

We extend our previous discussion by analyzing the optimal design of taxes and transfers in a rich dynamic quantitative environment with endogenous self-insurance. In particular, we incorporate a flexible tax-and-transfer system into a canonical heterogeneous-agent model (Aiyagari 1994) augmented with realistic labor income risk. We describe the model environment and its calibration in this section and discuss the optimal tax plan and the role of transfers in Section 4.

#### 3.1 Environment

The economy is populated by a continuum of households, a representative firm, and a government. The firm produces output by combining labor and capital, both of which are supplied by households. The government finances transfers and spending by taxing households' labor and capital incomes. We present the economy at its stationary equilibrium but will consider transitions when evaluating tax reforms.

*Households.*—Households value consumption  $c$  and leisure  $1 - n$ . Their idiosyncratic labor productivity  $z$  follows a Markov process with transition probabilities  $\pi_z(z', z)$ . Labor productivity shocks are uninsurable: households can only trade a one-period risk-free bond to self-insure, subject to a no-borrowing limit. Let  $V(a, z)$  be the maximal attainable value to a household with assets  $a$  and idiosyncratic productivity  $z$ :

$$\begin{aligned}
 V(a, z) = \max_{c, a', n} & \left\{ \frac{c^{1-\sigma}}{1-\sigma} - B \frac{n^{1+\varphi}}{1+\varphi} + \beta \mathbb{E}_{z'} [V(a', z') | z] \right\} \\
 \text{s.t.} & \\
 c + a' & \leq wzn + (1+r)a - \mathcal{T}(wzn, ra) \\
 a' & \geq 0,
 \end{aligned} \tag{12}$$

where  $w$  and  $r$  stand for wages and the interest rate, respectively. Households' taxes and transfers are captured by  $\mathcal{T}(wzn, ra)$ , which depend on labor income  $wzn$  and capital earnings  $ra$ . We discuss the shape of  $\mathcal{T}(\cdot)$  in detail later. Let  $n(a, z)$ ,  $c(a, z)$  and  $a'(a, z)$  denote a household's optimal policies.

*Representative firm.*—The representative firm demands labor and capital in order to maximize current profits

$$\Pi = \max_{K, L} \{ K^{1-\alpha} L^\alpha - wL - (r + \delta)K \}, \tag{13}$$

where  $\delta$  is the depreciation rate of capital. Optimality conditions for the firm are standard: marginal products are equalized to the cost of each factor.

*Government.*—The government’s budget constraint is given by:

$$G + (1 + r)D = D + \int \mathcal{T}(wzn, ra)d\mu(a, z) \quad (14)$$

where  $G$  is government spending,  $D$  is government debt, and  $\mu(a, z)$  is the measure of households with state  $(a, z)$  in the economy.

*Stationary equilibrium.*—Let  $A$  be the space for assets and  $Z$  the space for productivity. Define the state space  $S = A \times Z$  and let  $\mathcal{B}$  be the Borel  $\sigma$ -algebra induced by  $S$ . A formal definition of the competitive stationary equilibrium for this economy is provided next.

A **competitive stationary equilibrium** for this economy is given by value function  $V(a, z)$  and policies  $\{n(a, z), c(a, z), a'(a, z)\}$  for the household; policies for the firm  $\{L, K\}$ ; government decisions  $\{G, D, \mathcal{T}\}$ ; a measure  $\mu$  over  $\mathcal{B}$ ; and prices  $\{r, w\}$  such that, given prices and government decisions: (i) households’ policies solve their problems and achieve value  $V(a, z)$ , (ii) the firm’s policies solve its problem, (iii) the government’s budget constraint is satisfied, (iv) the capital market clears:  $K + D = \int_{\mathcal{B}} a'(a, z)d\mu(a, z)$ , (v) the labor market clears:  $L = \int_{\mathcal{B}} zn(a, z)d\mu(a, z)$ , (vi) the goods market clears:  $Y = \int_{\mathcal{B}} c(a, z)d\mu(a, z) + \delta K + G$ , and (vii) the measure  $\mu$  is consistent with households’ policies:  $\mu(\mathcal{B}) = \int_{\mathcal{B}} Q((a, z), \mathcal{B})d\mu(a, z)$  where  $Q$  is a transition function between any two periods defined by  $Q((a, z), \mathcal{B}) = \mathbb{I}_{\{a'(a, z) \in \mathcal{B}\}} \sum_{z' \in \mathcal{B}} \pi_z(z', z)$ .

### 3.2 A flexible tax-and-transfer function

We endow the government with two fiscal tools capturing the key elements of the U.S.  $t\&T$  system: a nonlinear labor tax and targeted transfers. Conveniently, the overall  $t\&T$  system is characterized by a few parameters only, all of which have a clear economic intuition. Yet, the function allows for flexible—and potentially non-monotonic—shapes of the overall marginal  $t\&T$  rates, a feature often found desirable in the optimal taxation literature.

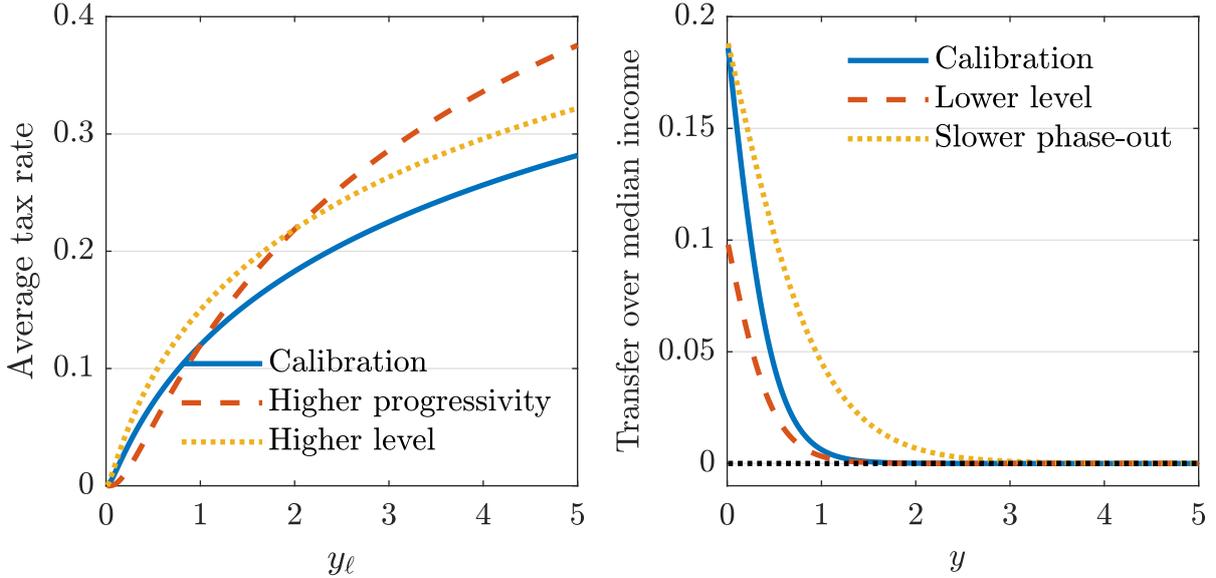
In particular, we divide the  $t\&T$  function  $\mathcal{T}(\cdot)$  into three components: a flat tax  $\tau_k$  on capital income  $y_k$ , a nonlinear tax  $\tau(y_\ell)$  on labor income  $y_\ell$ , and a targeted transfer component  $T(y)$  on total income  $y = y_k + y_\ell$ . From these components, we keep the capital tax constant (and purposely simple) and focus attention on labor taxes and transfers.<sup>9</sup>

We assume the average labor tax rate is characterized by two parameters,  $\theta$  and  $\lambda$ , as

$$\tau(y_\ell) = \exp \left( \log(\lambda) \left( \frac{y_\ell}{\bar{y}} \right)^{-2\theta} \right), \quad (15)$$

where  $\bar{y}$  is median income. As with the log-linear tax function used in Section 2, our

<sup>9</sup>For a recent detailed discussion on capital taxes, see [Boar and Midrigan \(2021\)](#).



**Figure 8:** Labor tax function and targeted transfers

**Notes:** The left panel illustrates the shape of the new labor tax function. It compares the average tax rate in the calibration ( $\lambda = 0.12$  and  $\theta = 0.16$ ) to a higher progressivity ( $\theta = 0.24$ ) and a higher level ( $\lambda = 0.15$ ). The right panel plots the new transfer function. It compares transfers normalized by median income in the calibration ( $m = 0.19$  and  $\xi = 4.1$ ) to a lower level ( $m = 0.1$ ) and a slower phase-out ( $\xi = 2$ ). Income is normalized by median income.

proposed tax function has two interpretable parameters:  $\theta$  for the progressivity and  $\lambda$  for its level. A positive (negative)  $\theta$  implies marginal tax rates that increase (decrease) with income. At  $\theta = 0$ , the tax is flat at  $\lambda$ . For all  $\theta$ , the tax rate is exactly  $\lambda$  when income is at its median,  $y_\ell = \bar{y}$ . The left panel of Figure 8 shows how  $\tau(y_\ell)$  varies with  $\theta$  and  $\lambda$ .

This labor tax function is always non-negative, in line with statutory tax rates in the United States. Other than that, our function largely resembles the log-linear tax used in Section 2, as shown in Figure C.1 of Appendix C.1. The level of progressivity  $\theta$  is approximately on the same scale as the progressivity  $\tau$  of the log-linear tax. The non-negative labor taxes imply that we rely exclusively on income-dependent transfers to generate negative  $t\&T$  rates, as we explain next.

We assume a transfer function that is characterized by two parameters: a level  $m$  and a phase-out rate  $\xi$ . In particular, the transfer given to a household with total income  $y$  is given as

$$T(y) = m\bar{y} \frac{2 \exp \left\{ -\xi \left( \frac{y}{\bar{y}} \right) \right\}}{1 + \exp \left\{ -\xi \left( \frac{y}{\bar{y}} \right) \right\}}. \quad (16)$$

The parameter  $m$  measures transfers to a household with zero income as a multiple of median income  $\bar{y}$ . The parameter  $\xi$  determines how quickly transfers phase-out with total income. When  $\xi = 0$ , transfers are a lump sum. As  $\xi$  becomes larger, transfers phase-out faster. The right panel of Figure 8 shows how transfers vary with  $m$  and  $\xi$ .

This functional form for transfers is a realistic description of U.S. income security programs, where transfers are means-tested, typically on both labor and capital income. Moreover, the phasing out of the transfer allows for non-monotonic marginal rates of the entire  $t&T$  system, which is not possible with a lump-sum transfer.

Thus, given the capital tax  $\tau_k$ , a government’s policy is characterized by four parameters: the progressivity and level of labor taxes,  $\theta$  and  $\lambda$ , and the level and phase-out rate of transfers,  $m$  and  $\xi$ . We refer to a fiscal plan  $\tau = \{\theta, \lambda, m, \xi\}$  as a government’s policy that satisfies its budget constraint (14).

### 3.3 Calibration

We calibrate the model to the U.S. economy in 2013. We take a period in the model to be a year. Several parameters are calibrated within the model, while others are taken from the literature, as we discuss next.

*Labor income risk.*—Recent empirical work has unveiled key statistics about labor income dynamics, two of which are potentially important for our purposes. First, the recent work in [Güvenen, Karahan, Ozkan, and Song \(2021\)](#) shows that earnings growth rates are negatively skewed and exhibit excess kurtosis. That is, relative to a normal distribution, there are more individuals with small and large earnings changes, but fewer with medium-sized earnings changes. Often, the large earnings changes are negative. Second, labor income inequality has increased in recent decades.<sup>10</sup> These facts are relevant to the efficiency and redistribution tradeoffs we analyze in this paper. Thus, we propose a process for idiosyncratic labor risk that can account for them.

In particular, we assume that a household’s productivity  $z$  follows a Gaussian Mixture Autoregressive (GMAR) process in logs as,

$$\begin{aligned} \log z_t &= \rho \log z_{t-1} + \eta_t \\ \eta_t &\sim \begin{cases} \mathcal{N}(\mu_1, \sigma_1^2) & \text{with probability } p_1, \\ \mathcal{N}(\mu_2, \sigma_2^2) & \text{with probability } 1 - p_1, \end{cases} \end{aligned} \quad (17)$$

with  $\mathbb{E}[\eta] = p_1\mu_1 + (1 - p_1)\mu_2 = 0$ , so that  $\mu_2$  is pinned down given  $\mu_1$  and  $p_1$ .

Most times, households draw an innovation from the first normal that has a low variance. Draws from the second normal occur infrequently, but they have a large variance and a negative mean. In this way, we can generate frequent small and infrequent large negative earnings changes—that is, the negative skewness and excess kurtosis empirically documented for labor income growth. We discretize the productivity process using the method of [Farmer and Toda \(2017\)](#).

Furthermore, to better capture the concentration of incomes at the top, we follow

---

<sup>10</sup>See [Piketty and Saez \(2003\)](#).

Hubmer, Krusell, and Smith (2020) and make one additional adjustment to the productivity process. We adjust the top 15% states in our productivity grid such that they follow a Pareto distribution with tail  $\kappa = 1.6$  as estimated by Aoki and Nirei (2017).

The income process is then characterized by five parameters:  $(\rho, \mu_1, \sigma_1, \sigma_2, p_1)$ . We pick these parameters to match key statistics of households' labor earnings growth from the Panel Study of Income Dynamics (2021) (PSID) as well as income concentration at the top.<sup>11</sup> We target four moments of the labor income growth distribution: the standard deviation (0.33), the difference between the ninetieth and the tenth percentile (0.61), the skewness (-0.35), and the kurtosis (11.43). Additionally, we target a labor income share of 38% for the top 10% of labor income earners, as found in the Survey of Consumer Finances for 2013. Table 1 contains all the parameter values.

*Taxes and transfers.*—We set capital taxes at 35% (Trabandt and Uhlig 2011) and calibrate the four parameters  $\tau$  of the fiscal function to match the tax-and-transfer rates from the CBO shown in Figure 1. We target the distribution of transfer rates across households and calibrate the transfer level at  $m = 0.19$  with a rapid phase-out at  $\xi = 4.1$ . As such, the lowest-income household receives \$10,700 in 2013 dollars. For labor taxes, we calibrate the level at  $\lambda = 12\%$  and progressivity at  $\theta = 0.16$  to target average tax rates in the second and fifth quintile. As Table 2 shows, the model matches reasonably well the distribution of transfers and income-tax rates across households.<sup>12</sup> Figure 9 plots average and marginal  $t$  &  $T$  rates as a function of labor income for different values of capital income.

*Remaining parameters.*—We set the coefficient of relative risk aversion  $\sigma$  to 2, a value that is more standard in quantitative macroeconomics than the log utility used for tractability in Section 2. We fix the Frisch elasticity  $\varphi$  to 0.4, also a common value. We set the production side parameters to standard values, with the labor share  $\alpha = 0.64$  and the (annual) depreciation rate  $\delta = 0.08$ . We calibrate the discount factor  $\beta$ , government debt  $D$ , and labor disutility  $B$  to jointly match an interest rate of 2%, a government debt-to-output ratio of 60%, and an average labor supply of 0.3. Spending is implied by government budget clearing and results in a spending-to-output ratio of 13%, in line with the data. All parameters are summarized in Table 1.

*Distributions.*—The model matches well the distribution of income, as Table 3 shows. While the labor-income share of the top 10% is targeted, the model features a remarkable fit for all income quintiles. Importantly, the model captures accurately how poor the poor are: labor income at the first, fifth and tenth percentiles amounts to 10%, 22%, and 31% of median labor income, vs. 10%, 20%, and 31% in the data. The lowest-income household

<sup>11</sup>We do not directly use the statistics reported in Guvenen, Karahan, Ozkan, and Song (2021), because they are computed at the individual level. Instead, we follow De Nardi, Fella, and Paz-Pardo (2019) and use PSID data to compute income growth statistics at the household level, but base estimates on pre-tax income data. See Appendix B.2 for more details.

<sup>12</sup>See Appendix B.1 for details on computations of transfers and tax rates in the data.

**Table 1:** Parameter values

<b>Preferences</b>			<b>Income Process</b>		
$\beta$	Discount factor	0.962	$\rho$	Persistence	0.935
$\sigma$	Risk aversion	2.000	$p_1$	Weight on first normal	0.850
$1/\varphi$	Labor supply elasticity	0.400	$\mu_1$	Mean of first normal	0.016
$B$	Disutility of labor	85.00	$\mu_2$	Mean of second normal	-0.091
<b>Production</b>			$\sigma_1$	Std. dev. of first normal	0.150
$\delta$	Depreciation rate	0.080	$\sigma_2$	Std. dev. of second normal	0.630
$\alpha$	Labor share	0.640	$\kappa$	Pareto tail parameter	1.600
<b>Government</b>					
$\theta$	Tax progressivity	0.160	$D$	Public debt	0.600
$\lambda$	Tax level	0.118	$G$	Government spending	0.126
$m$	Transfer level	0.190	$\tau_k$	Capital tax rate	0.350
$\xi$	Transfer phase-out	4.100			

**Table 2:** Calibration: taxes and transfers

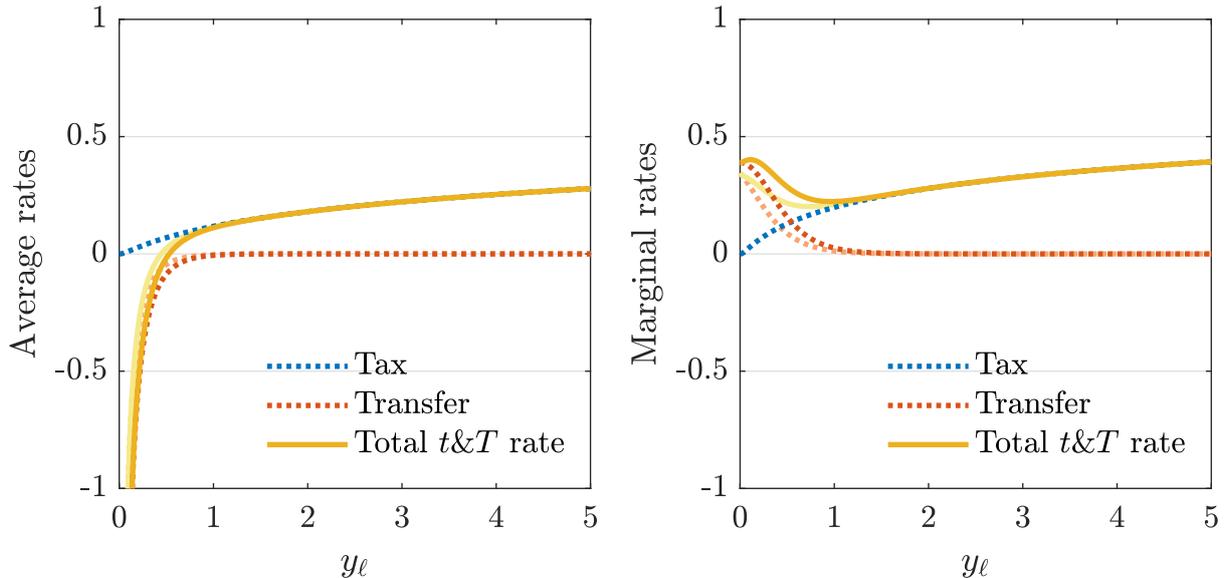
<b>Tax rates</b>	Q1	Q2	Q3	Q4	Q5
Data	0%	10%	16%	20%	27%
Model	8%	11%	14%	17%	28%
<b>Transfers rates</b>	Q1	Q2	Q3	Q4	Q5
Data	26%	3%	1%	0%	0%
Model	24%	4%	1%	0%	0%

**Notes:** Average tax and transfers rates per income quintile. Data: CBO 2013, working-age households.

**Table 3:** Calibration: income and wealth

<b>Labor income</b>	Q1	Q2	Q3	Q4	Q5	Top 10
Data	4%	9%	14%	21%	53%	38%
Model	4%	9%	14%	20%	52%	38%
<b>Net worth</b>	Q1	Q2	Q3	Q4	Q5	Top 10
Data	-1%	1%	3%	10%	87%	74%
Model	0%	2%	8%	18%	72%	52%

**Notes:** Labor income shares by labor income quintiles and wealth shares by wealth quintiles. Data: SCF 2013, households aged 25-60.



**Figure 9:** Calibration: average and marginal tax-and-transfer rates

**Notes:** This figure shows average and marginal  $t&T$  rates in the calibrated steady state along the labor income distribution, normalized by median income. As transfers phase-out with respect to total income, the level of capital income matters for the  $t&T$  rates. We report two cases: zero capital income (legend color) and mean capital income (lighter color).

earns exactly \$5,000 in the model, in line with our data earning threshold. As we show in Section 5, the left and right tails of the income distribution shape the optimal plan. In contrast, the model falls short in generating enough wealth concentration at the top, a common shortcoming of this type of model. Section 5 considers a model extension with heterogeneous stochastic discount factors to improve the wealth distribution fit.

### 3.4 Wealth effects and elasticities

Wealth effects on labor supply are key to the optimal design of the  $t&T$  system. Jointly with the Frisch elasticity, wealth effects determine the response of labor supply to a tax change. Golosov, Graber, Mogstad, and Novgorodsky (2021) estimates these wealth effects by combining U.S. taxpayers' income data with data on lottery winnings, a plausibly exogenous variation in wealth. We replicate their estimates in our model which we report in Table 4.<sup>13</sup>

The model wealth effects are in line with the estimates in Golosov et al. (2021). We report wealth effects for an average and a large win size, the latter being more representative of the magnitude of transfers we analyze in Section 4.<sup>14</sup> The estimates represent the five-year average change in labor income for every \$100 won, as reported in Golosov

<sup>13</sup>See Appendix C.2 for computations of wealth effects and elasticities.

<sup>14</sup>As explained in Appendix C.2, we compute the present discounted value of transfers based on the calibrated after-tax interest rate.

**Table 4:** Wealth effects: model and data

<b>Average win</b>	All	Q1	Q2	Q3	Q4
Data	-2.3	-1.3	-2.3	-2.6	-3.1
Model	-2.6	-2.4	-2.7	-2.6	-2.5
<b>Large win</b>	All	Q1	Q2	Q3	Q4
Data	-0.9	-	-	-	-
Model	-1.2	-0.7	-1.1	-1.3	-1.7

**Notes:** Wealth effects in model and data as reported in Golosov et al. (2021). The model estimates represent the five-year average change in labor income for every \$100 won. Quartiles are defined by pre-win income. We use an average win size of \$180,000 for the model, corresponding to the average size of the lottery win in the treatment group in Golosov et al. (2021). The large win size correspond to 2M.

et al. (2021). For the average win size, labor income declines \$2.3 in the data and \$2.6 in the model. For the large win size, the declines are \$0.9 in the data and \$1.2 in the model. The estimated wealth effect declines with pre-win income, a feature the model also captures for the large win size. If at all, the model slightly overestimates wealth effects, which may lead to understating the optimal size of transfers in Section 4.

Finally, the model also generates a reasonable labor elasticity at the top. We conduct a partial-equilibrium experiment in which marginal tax rates increase by 1% unexpectedly and permanently, as in the tax reform we consider next. The implied labor elasticity of the top 1% is 0.20 in the model, a number well within the range of values reported in the literature.<sup>15</sup>

## 4 Optimal tax-and-transfer plan

In this section, we show that the findings of the analytical model carry over to our quantitative environment: a planner optimally trades higher transfers for lower income-tax progressivity. When including realistic efficiency and redistribution concerns, the optimal fiscal plan features generous transfers and moderate labor tax progressivity. Thus, as in the analytical case, transfers are key to generate more progressive average than marginal  $t$  &  $T$  rates. Furthermore, we show that while a phasing-out of transfers is optimal, a lump-sum transfer comes close in terms of welfare gains.

<sup>15</sup>See Kindermann and Krueger (2021) for a discussion of the empirical literature and the model counterpart in a related framework.

## 4.1 A Ramsey approach

A government’s plan is fully characterized by  $\tau = \{\theta, \lambda, m, \xi\}$ , the progressivity and level of labor taxes, and the level and phase-out rate of transfers. We use a utilitarian welfare criterion to evaluate a one-time change in policy  $\tau$  and include transitions in the welfare computations.<sup>16</sup>

In particular, let  $V_0(a, z; \tau)$  be the life-time utility of a household with assets  $a$  and productivity  $z$  in the period when the policy  $\tau$  is implemented. The utilitarian welfare criterion  $\mathcal{W}(\tau)$  considers the sum of utilities  $V_0(\cdot)$  as,

$$\mathcal{W}(\tau) = \int V_0(a, z; \tau) d\mu_0(a, z). \quad (18)$$

Notice that policy  $\tau$  affects household life-time utility, but the measure  $\mu_0(a, z)$  is given by the initial steady state of the economy.<sup>17</sup>

## 4.2 Optimal tax-and-transfer system

The optimal system is substantially more redistributive than the system currently in place in the United States. Optimal transfers are large, at \$29,100 in 2013 U.S. dollars for the lowest-income household. This value implies an income floor of 45% of median income ( $m = 0.45$ ). The optimal phasing-out is slow—at  $\xi = 1.85$  compared to  $\xi = 4.1$  in the calibration—implying a transfer of \$8,200 for a household at calibrated median income. Optimal income-tax progressivity is moderate at  $\theta = 0.16$ , similar to its calibrated value.

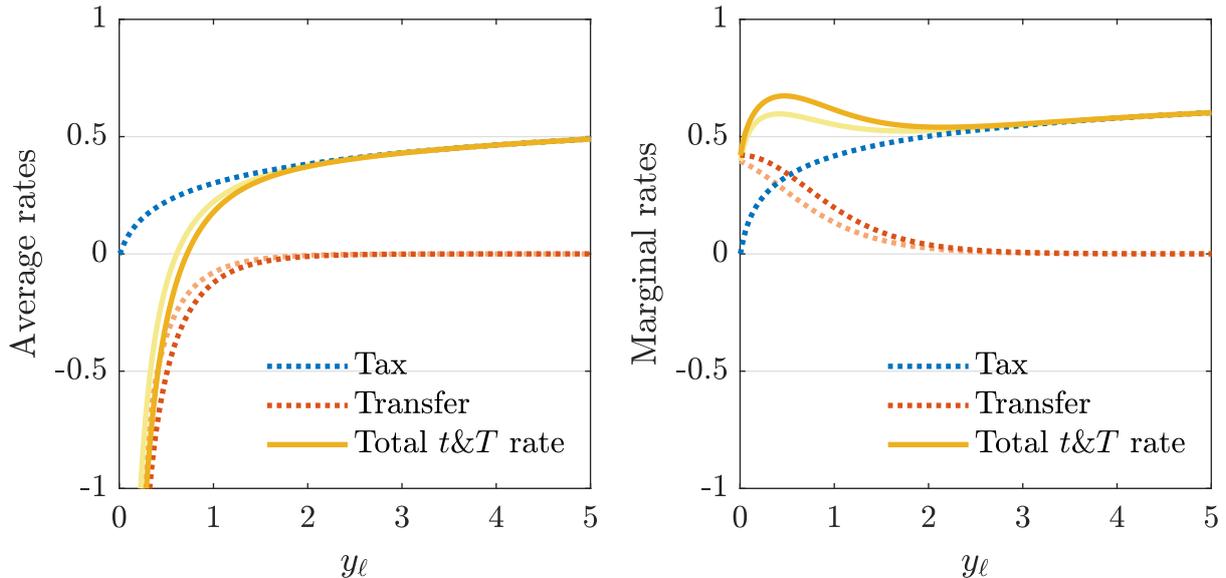
Optimal average  $t\&T$  rates are more progressive than marginal rates, as Figure 10 shows. Average rates monotonically increase with income, because of both transfers and progressive labor taxes. Marginal rates, however, are not monotonic. They are high for low-income earners because transfers phase out, lower for medium-income earners, and high again for top-income earners because of progressive labor taxes.

Transfers in the optimal system are substantially more generous than in the status quo. As shown in Table 5, the transfer rate is 171% for the bottom income quintile, compared to only 26% in the data. Transfers also remain generous for the second and third quintiles in the optimal plan, while they are virtually zero in the empirical counterpart. The larger transfers are financed with higher labor taxes, but the optimal income-tax progressivity is similar to the status quo. Thus, tax rates increase roughly uniformly across quintiles. Table 5 also shows average and marginal  $t\&T$  rates by income quin-

---

<sup>16</sup>We actually optimize on three parameters— $\theta$ ,  $\lambda$ , and  $\xi$ —and set  $m$  to satisfy the budget constraint. Thus,  $m$  varies somewhat along the transition. We report the long-run value of  $m$ . See Appendix C.3 for computational details. For optimal time-varying tax systems, see Acikgöz, Hagedorn, Holter, and Wang (2021) and Dyrda and Pedroni (2021).

<sup>17</sup>Appendix C.4 reports the optimal fiscal plan when optimizing on steady-state welfare.



**Figure 10:** Optimal tax-and-transfer system: average and marginal  $t&T$  rates

**Notes:** This figure shows optimal average and marginal  $t&T$  rates along the labor income distribution, normalized by calibrated median income. We report two cases: zero capital income (legend color) and mean capital income (lighter color).

tile. Average  $t&T$  rates are more progressive than in the status quo: they are equal to  $-155\%$  for the bottom quintile and monotonically increase with income to reach  $44\%$  for the top quintile. Marginal  $t&T$  rates are non-monotonic, at above  $60\%$  in the bottom three quintiles and around  $50\%$  in the top two quintiles.

Overall, the optimal plan achieves redistribution via large transfers at the bottom. It also preserves efficiency with moderate income-tax progressivity, as to incentivize labor from productive households. This quantitative finding is in line with the analytical model, where transfers are optimally used to disentangle average and marginal progressivity.

### 4.3 The economy shrinks, but most people benefit

The optimal tax-and-transfer system results in a decline in economic activity, as Figure 11 shows. The higher taxes and more generous transfers lead to lower savings, and thus wages decline while the interest rate increases. The distribution of hours worked in the final steady state also shifts left, as Figure 12 shows. Overall, output in the final steady state is  $18\%$  lower than in the initial steady state.

Yet, not only does the optimal tax-and-transfer system increase utilitarian welfare, but it is also favored by a majority of households over the status quo. Aggregate welfare gains amount to  $9.64\%$  in consumption equivalent terms.<sup>18</sup> As Figure 13 shows, these large welfare gains accrue primarily to the poor, who benefit the most from the generous

<sup>18</sup>See Appendix C.5 for a formal derivation of consumption equivalents in our environment.

**Table 5:** Optimal tax-and-transfer system

<b>Data</b>	Q1	Q2	Q3	Q4	Q5
Tax rate	0%	10%	16%	20%	27%
Transfer rate	26%	3%	1%	0%	0%
Average $t&T$ rate	-26%	-7%	15%	20%	27%
<b>Optimal plan</b>	Q1	Q2	Q3	Q4	Q5
Tax rate	16%	22%	27%	32%	44%
Transfer rate	171%	59%	22%	6%	0%
Average $t&T$ rate	-155%	-37%	5%	26%	44%
Marginal $t&T$ rate	61%	66%	62%	54%	51%
<b>UBI plan</b>	Q1	Q2	Q3	Q4	Q5
Tax rate	56%	56%	57%	55%	58%
Transfer rate	180%	85%	53%	35%	13%
Average $t&T$ rate	-124%	-29%	4%	20%	46%
Marginal $t&T$ rate	60%	61%	62%	63%	64%

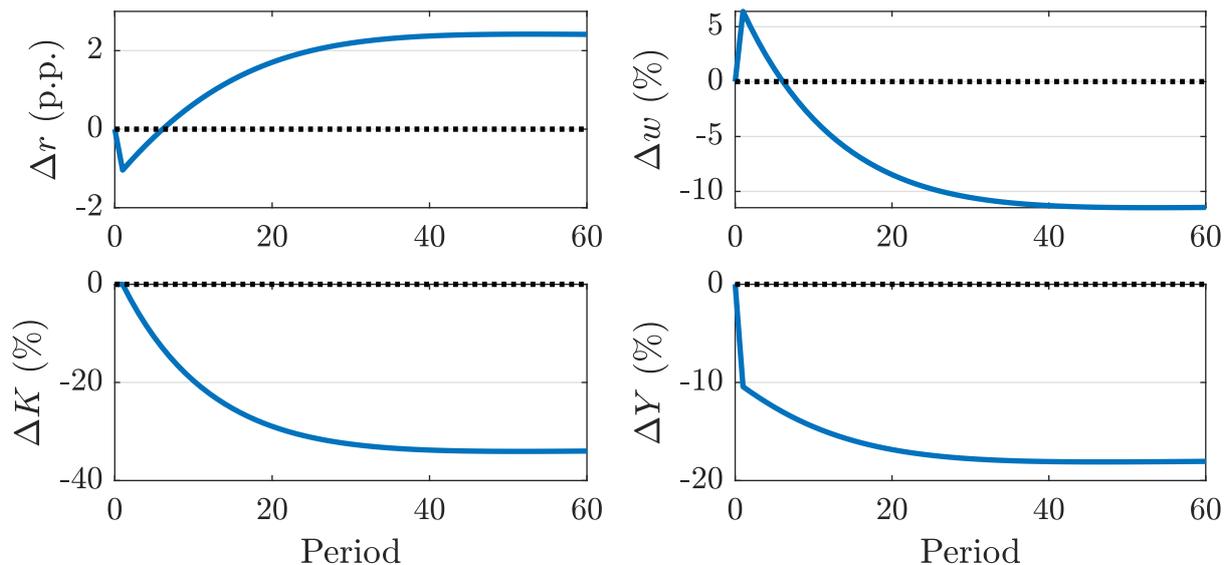
**Notes:** This table shows average tax and transfer rates, as well as average and marginal  $t&T$  rates, by income quintile. It reports the CBO data, the optimal plan with targeted transfers, and the UBI plan with lump-sum transfers. The CBO does not report marginal rates.

transfers. Households with high productivity experience welfare losses on average, though those with higher assets may still benefit from the tax reform due to the higher interest rates. Overall, 79% of households benefit from implementing the optimal plan.

We follow [Bhandari, Evans, Golosov, and Sargent \(2021\)](#) and decompose the welfare gains into three components: aggregate efficiency, redistribution, and insurance. Efficiency captures the welfare gains resulting from changes in aggregate resources. Redistribution captures changes in ex-ante shares of consumption and leisure, while insurance captures changes in ex-post utility risk.<sup>19</sup>

About two-thirds of the gains come from insurance, one-fourth from redistribution and the remainder from the efficiency component. The insurance gains reflect lower volatility of consumption due to larger transfers. All households record a gain in the insurance component, but this gain is larger for low-asset households. The redistribution component is driven by lower dispersion in ex-ante consumption shares. This component is heterogeneous across households: it is large for low-asset/low-productivity households, while it is negative for high-productivity but low-asset households, in line with the distribution of consumption equivalents. Overall, the redistribution component masks substantial heterogeneity, which aggregates to a positive but small number. Interestingly, the efficiency

<sup>19</sup>See Appendix C.5 for more details.



**Figure 11:** Quantitative model: responses of prices and quantities to the tax reform

**Notes:** This figure plots the transition path for the interest rate, wages, capital and output after the tax reform is implemented.  $t = 0$  shows the calibrated steady state. Responses of wages, output and capital are plotted in percentage deviation from steady state; the interest rate response is plotted in differences.

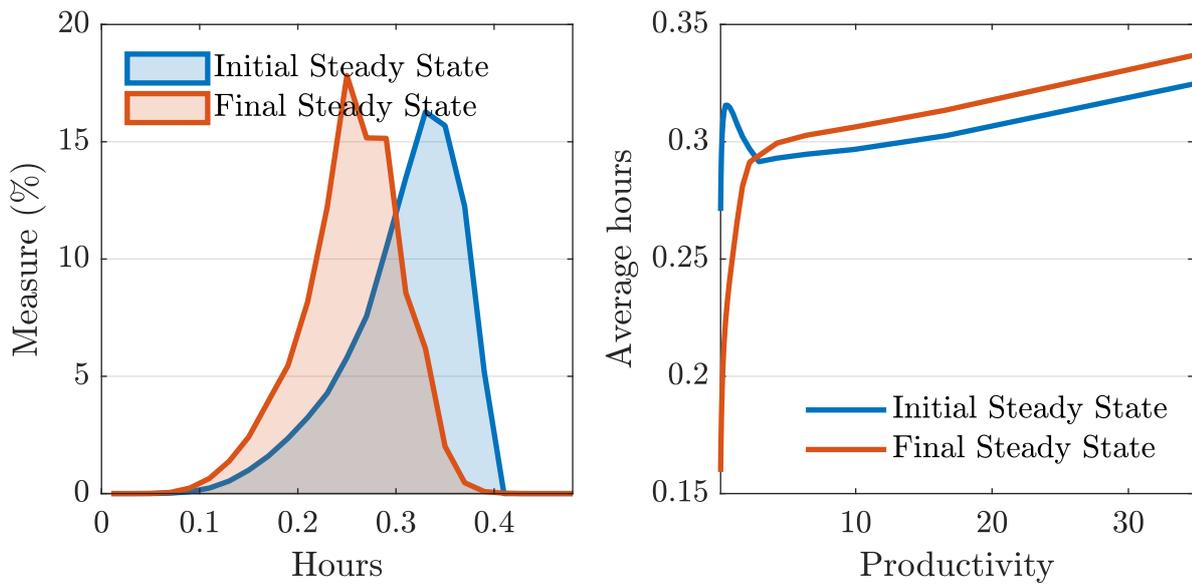
component is slightly positive, as the decrease in aggregate consumption is offset by larger leisure. Efficiency gains are driven by a better allocation of hours worked, as shown in the right panel of Figure 12.

#### 4.4 Exploring the phase-out of transfers: UBI and affine plans

A positive phase-out of transfers allows to implement non-monotonic marginal  $t&T$  rates, which is a property featured in the optimal plan and is also reminiscent of the optimal U-shaped marginal rates often found in the public finance literature. To evaluate the importance of non-monotonic marginal rates, we compute the optimal plan when transfers do not phase-out ( $\xi = 0$ ). In this case, the shape of marginal  $t&T$  rates is monotonic and determined by the labor tax progressivity.

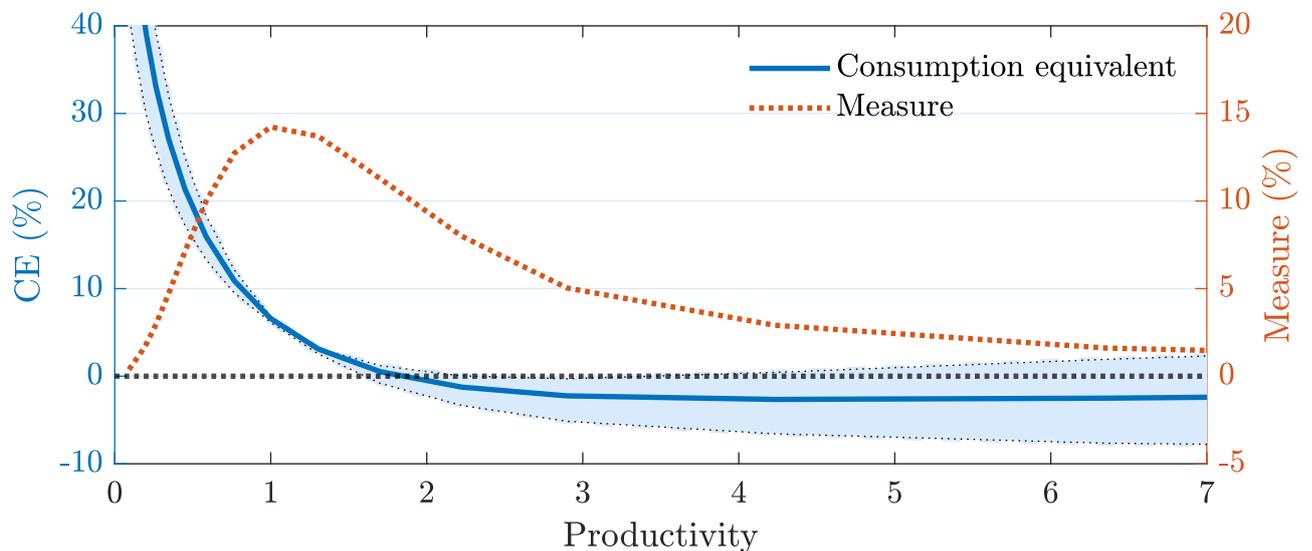
Eliminating the transfer phase-out allows us to discuss two common tax proposals: a universal basic income (UBI) and an affine plan. The UBI plan eliminates the phase-out of transfers but still optimizes the progressivity of labor taxes  $\theta$ . The affine plan is a UBI plan financed with flat income taxes—that is,  $\theta = 0$ .

The UBI plan includes large transfers at  $m = 0.43$ , about \$28,800 for each household, which are optimally financed with almost flat labor taxes at  $\theta = 0.03$ . Transfers in the optimal UBI amount to a large government outlay, representing 26% of GDP compared to 10% in the benchmark plan. In order to finance the transfers and preserve labor supply incentives, income-tax progressivity falls as to maintain roughly flat marginal



**Figure 12:** Quantitative model: hours worked in the initial and final steady states

**Notes:** The left panel plots the distribution of hours worked in the initial and final steady states. The right panel shows average hours worked by productivity level in the initial and final steady states.



**Figure 13:** Optimal tax-and-transfer system: consumption equivalent welfare gains

**Notes:** This figure shows welfare gains, in terms of consumption equivalent (CE), from a tax reform to the optimal system. The solid line plots the average CE by productivity level  $z$ , while the two dashed lines show the bottom-20% and the top-80% of the distribution of CE at each  $z$ . Both axes are cut for readability; welfare gains peak at 56% for the lowest-productivity level and converge to about -2% for top levels of productivity. The right axis plots the measure of households for each productivity level.

$t&T$  rates. Labor taxes are thus higher than in the plan with phase-out and almost constant across households, spanning from 60% in the first income quintile to 64% in the top income quintile. Although with different tax and transfer rates, the UBI plan achieves about the same level of redistribution as with phase-out, with comparable average  $t&T$  rates across households—see Table 5.

The optimal affine plan is similar to the UBI, as the latter uses almost flat taxes. The affine plan features a lump-sum transfer of \$30,000 to each household and a tax rate of 64%.

The welfare gain of the UBI is 9.43% in consumption equivalent terms, while the affine gains are 9.38%, both close to the gains in the plan with phase-out. Thus, our framework is supportive of lump-sum transfers, to the extent that tax rates above 60% are implementable. Otherwise, a planner should favor a phase-out of transfers, which is associated with lower tax rates.

Overall, the UBI and affine exercises point to the importance of disentangling average and marginal rates. Transfers, even if lump-sum, allow to separate the progressivity of average and marginal tax rates. The phasing-out of transfers additionally allows for non-monotonic marginal tax rates. The exercises in this section suggest that disentangling the progressivity of average and marginal tax rates can generate most of the welfare gains, while phasing-out of transfers allows for lower income-tax rates.

*Comparison to the optimal log-linear plan.*—We compute the optimal plan using a log-linear labor tax function as in the analytical section.<sup>20</sup> The optimal plan features large progressivity, at  $\tau = 0.396$  when maximizing steady-state welfare and  $\tau = 0.490$  when including transitions, compared to the estimated  $\tau = 0.18$  for the current U.S. system.<sup>21</sup> Yet, compared to the optimal plan with targeted transfers, the optimal log-linear plan achieves less redistribution and at the cost of strongly increasing marginal  $t&T$  rates (see Table C.3). Welfare gains are large, at +5.08%, but smaller than with targeted transfers, in particular for households at the bottom of the income distribution.

In contrast with Heathcote and Tsujiyama (2021), we find that the affine plan is preferable to the log-linear plan. This difference may be driven by several factors, including endogenous private insurance, which reinforces the value of insuring consumption fluctuations; capital accumulation, which may lower optimal marginal rates at the top to preserve capital accumulation incentives; and higher risk-aversion, which increases the welfare gains associated with a lump-sum transfer.<sup>22</sup>

<sup>20</sup>See Appendix C.6 for more details.

<sup>21</sup>Heathcote and Tsujiyama (2021) find an optimal  $\tau = 0.33$  when maximizing steady-state welfare.

<sup>22</sup>See Chang and Park (2021) for the effects of private insurance on marginal rates in a dynamic Mirrleesian set-up. See Heathcote and Tsujiyama (2021), Table 2, for the optimal lump-sum in the static Mirrleesian plan as a function of risk-aversion.

## 5 Trading off progressivity and transfers

A key insight of the analytical model is the optimal negative relation between transfers and progressivity. This relation remains valid in the quantitative model: for a given phase-out  $\xi$ , larger transfers  $m$  are optimally associated with lower progressivity  $\theta$ . We refer to this relation as the  $m$ - $\theta$  line. We show that the optimal  $m$ - $\theta$  line is robustly downward-sloping and explore how it shifts with the distributions of income, wealth, and income risk.

We compute the optimal  $m$ - $\theta$  line for a given  $\xi$ . To ease exposition, we assume no phase-out ( $\xi = 0$ ) throughout this section, but results are robust to using positive  $\xi$ .<sup>23</sup>

### 5.1 Income distribution: a story of two tails

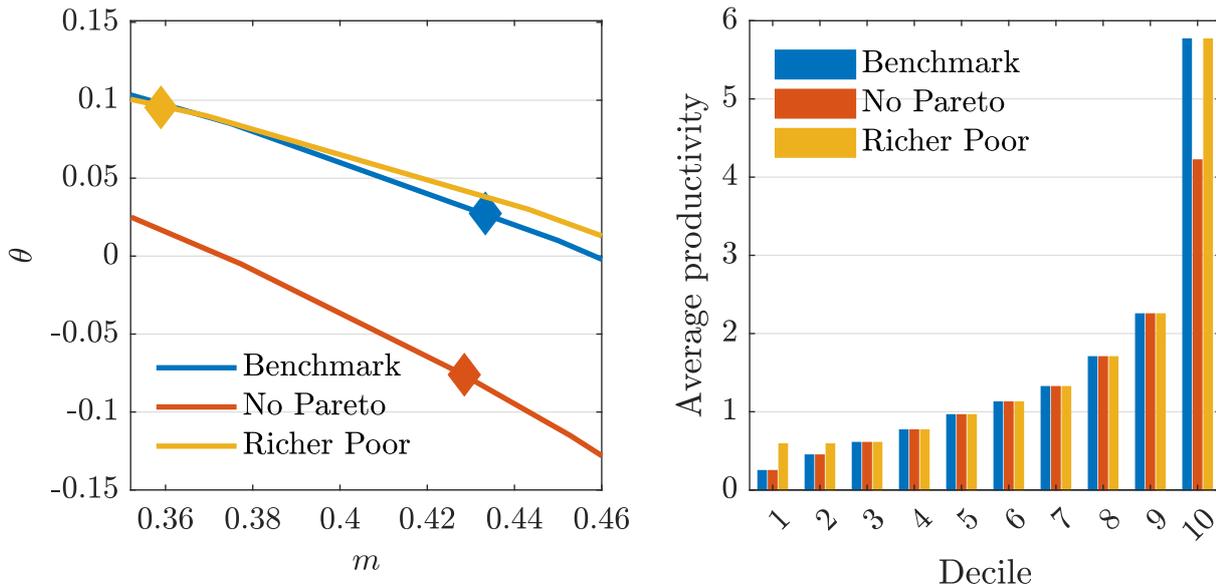
The downward-sloping  $m$ - $\theta$  line shifts with the left and right tails of the income distribution. To show this, we compute the  $m$ - $\theta$  line in our benchmark and in another two economies: a “No Pareto” economy and a “Richer Poor” economy. For the “No Pareto” economy, we remove the Pareto tail adjustment to the highest values of the productivity grid. For the “Richer Poor”, we increase the lowest 20% of the productivity distribution to equate its 20<sup>th</sup>-percentile. In turn, the 10<sup>th</sup>-percentile of the labor-income distribution amounts to 50% of the median labor income, compared to 31% in the benchmark economy. In both cases, we readjust all remaining parameters to match the same calibration targets as in our benchmark, except for the income distribution. Figure 14 shows the optimal  $m$ - $\theta$  relation for the benchmark and the two additional cases, as well as the productivity distribution in the three cases.

In the “No Pareto” economy, the  $m$ - $\theta$  line shifts down. That is, for a given level of transfer  $m$ , the optimal progressivity  $\theta$  falls significantly, by about 0.10. This finding confirms Mankiw, Weinzierl, and Yagan (2009) and Heathcote and Tsujiyama (2021), which emphasize the importance of the right tail of the income distribution in determining the optimal level of progressivity. Remarkably, the optimal level of transfers  $m$  is almost the same as in the benchmark economy, but the optimal plan features regressive income taxes, as in the simple model of Section 2. Thus, the concentration of income at the top does not significantly affect the size of optimal transfers, but it does change the optimal way of financing them. With the Pareto tail, a planner is able to raise sufficient revenues from the top and prefers lower tax rates on middle-income households.

In contrast, the  $m$ - $\theta$  line barely shifts in the “Richer Poor” economy. That is, for a given level of transfer  $m$ , the optimal level of progressivity is similar to the benchmark. However, optimal transfers decrease to  $m = 0.36$  compared to  $m = 0.43$  in the benchmark, a drop of about \$4,900. Thus, the lower the income of the poorest is, the

---

<sup>23</sup>See Appendix C.7 for results with  $\xi > 0$ .



**Figure 14:** Optimal  $m$ - $\theta$  line and the income distribution

**Notes:** The left panel plots the optimal progressivity  $\theta$  for each level of transfer  $m$ , as well as the optimal  $(m, \theta)$  pair, for three economies: the benchmark, the “No Pareto”, and the “Richer Poor” economies. The phase-out parameter  $\xi$  is fixed to zero. The right panel shows the average productivity level  $z$  by decile in each economy.

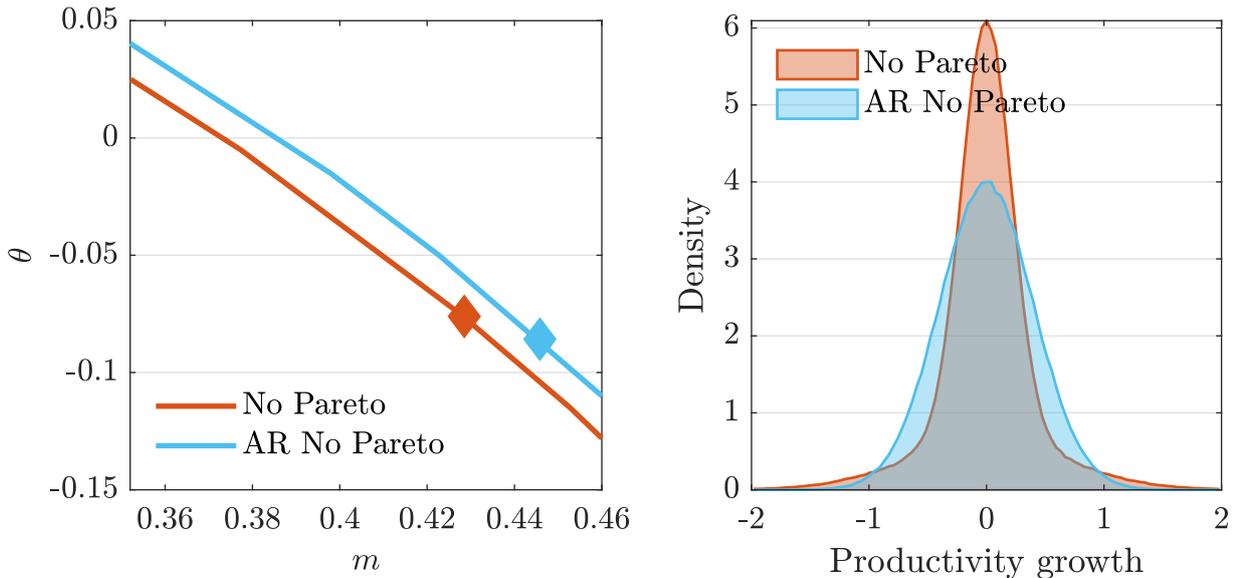
larger the transfers the optimal plan prescribes.

## 5.2 Income risk distribution: skewness and kurtosis

Higher-order moments of the labor income distribution only moderately affect the optimal fiscal plan. We compute the  $m$ - $\theta$  line for the case when labor productivity follows an AR(1) process, abstracting from the Pareto tail for simplicity. We label this case as “AR No Pareto” and compare it to the “No Pareto” economy, which features the GMAR process described in Section 3. We keep the persistence of the AR(1) fixed to the calibrated value and set its innovation variance to match the same overall productivity variance as in the GMAR case. The left panel of Figure 15 shows the optimal  $m$ - $\theta$  line for the “AR No Pareto” and “No Pareto” cases.

As compared to the GMAR, the AR process has fewer large negative shocks but more frequent medium-size shocks, as shown in the right panel of Figure 15. These differences have offsetting effects on optimal taxes: while the less frequent left tail shocks reduce the need for insurance, the more frequent medium-size shocks do the opposite. This results in a similar progressivity and somewhat larger transfers—from  $m = 0.43$  to  $m = 0.45$ , an increase of about \$1,200. Overall, higher order moments of income risk do not drastically change the optimal  $t$ & $T$  plan.<sup>24</sup>

<sup>24</sup>This result is consistent with De Nardi, Fella, and Paz-Pardo (2019), which finds that the welfare cost of idiosyncratic risk is somewhat lower when modeling higher-order moments of income risk.



**Figure 15:** Optimal  $m$ - $\theta$  line and the distribution of income risk

**Notes:** The left panel plots the optimal progressivity  $\theta$  for each level of transfer  $m$ , as well as the optimal  $(m, \theta)$  pair, for two economies: the “No Pareto” economy with the GMAR process, and the “AR No Pareto” with the AR process. The phase-out parameter  $\xi$  is fixed to zero. The right panel shows the distribution of productivity changes in both economies.

### 5.3 Wealth inequality

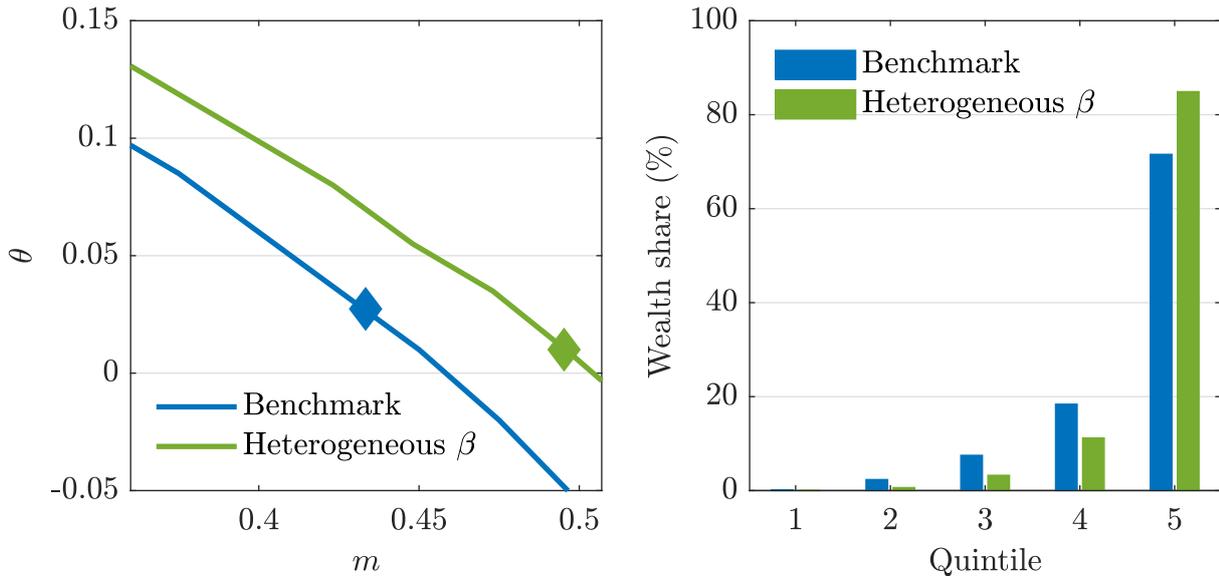
Higher wealth concentration reinforces redistribution concerns. We compute the  $m$ - $\theta$  line in a case with heterogeneous stochastic discount factors, as in [Krusell and Smith \(1998\)](#), which we label “Heterogeneous  $\beta$ ”. We calibrate the process for discount factors to match better the concentration of wealth at the top, a moment that the benchmark model could not replicate.<sup>25</sup> Figure 16 shows the optimal  $m$ - $\theta$  line in the “Heterogeneous  $\beta$ ” and in the benchmark as well as the wealth distribution in both cases.

The optimal  $m$ - $\theta$  line remains downward sloping but shifts up in the “Heterogeneous  $\beta$ ” case: for a given level of transfers, the optimal progressivity is larger, by about 0.05. With a higher wealth concentration, redistribution concerns lead a planner to finance transfers with more progressive income taxes, as high-wealth households are also likely to be high-income. Optimal transfers increase by \$4,100, from  $m = 0.43$  to  $m = 0.50$ , such that optimal progressivity  $\theta$  remains about the same as in the benchmark.

This exercise should be taken with caution, as wealth inequality is generated with a simple and, importantly, policy-invariant modeling tool.<sup>26</sup> Yet, it suggests that the benchmark calibration delivers a lower bound for the generosity of the optimal tax plan.

<sup>25</sup>See [Appendix C.7](#) for more details.

<sup>26</sup>See [Boar and Midrigan \(2021\)](#) and [Brüggemann \(2021\)](#) for a richer modeling of wealth accumulation due to entrepreneurial activity.



**Figure 16:** Optimal  $m$ - $\theta$  line and the wealth distribution

**Notes:** The left panel plots the optimal progressivity  $\theta$  for each level of transfer  $m$ , as well as the optimal  $(m, \theta)$  pair, for the benchmark and the “Heterogeneous  $\beta$ ” economies. The phase-out parameter  $\xi$  is fixed to zero. The right panel shows the distribution of wealth by quintile in both economies.

## 6 Conclusion

In this paper, we studied the optimal design of the tax-and-transfer system. We developed a tractable analytical model demonstrating an optimally negative relation between transfers and income-tax progressivity, due to both *efficiency* and *redistribution* concerns. We then quantified the optimal fiscal plan in a rich dynamic model calibrated to the U.S. economy. We found that optimal transfers should be generous, with a slow phase-out, and financed with moderate income-tax progressivity. That is, the optimal plan features more progressive average than marginal  $t&T$  rates. Furthermore, there are large welfare gains from implementing this plan, that would be supported by a majority of households.

Our Ramsey approach is compatible with a quantitative and dynamic evaluation of efficiency and redistribution concerns. The instruments we used are simple and intuitive, and they resemble current policies implemented by many countries. Yet, they are richer than what is typically used in the Ramsey literature, and flexible enough to generate nonlinear non-monotonic  $t&T$  schedules. As such, our analysis contributes to bridging the gap between Mirrlees and Ramsey traditions.

## References

Acikgöz, Ömer, Marcus Hagedorn, Hans Holter, and Yikai Wang (2021). “The Optimum Quantity of Capital and Debt.” *Working Paper*.

- Aiyagari, S. Rao (1994). “Uninsured idiosyncratic risk and aggregate saving.” *The Quarterly Journal of Economics* 109.3, pp. 659–684.
- Aoki, Shuhei and Makoto Nirei (2017). “Zipf’s Law, Pareto’s Law, and the Evolution of Top Incomes in the United States.” *American Economic Journal: Macroeconomics* 9.3, pp. 36–71.
- Arnoud, Antoine, Fatih Guvenen, and Tatjana Kleineberg (2019). “Benchmarking Global Optimizers.” *Working Paper*.
- Ayaz, Mehmet, Lea Fricke, Clemens Fuest, and Dominik Sachs (2021). “Who should bear the burden of COVID-19 related fiscal pressure? An optimal income taxation perspective.” *Working Paper*.
- Badel, Alejandro, Mark Huggett, and Wenlan Luo (2020). “Taxing top earners: a human capital perspective.” *The Economic Journal* 130.629, pp. 1200–1225.
- Bakiş, Ozan, Barış Kaymak, and Markus Poschke (2015). “Transitional dynamics and the optimal progressivity of income redistribution.” *Review of Economic Dynamics* 18.3, pp. 679–693.
- Benabou, Roland (2000). “Unequal societies: Income distribution and the social contract.” *The American Economic Review* 90.1, pp. 96–129.
- Bhandari, Anmol, David Evans, Mikhail Golosov, and Thomas J. Sargent (2021). “Efficiency, Insurance, and Redistribution Effects of Government Policies.” *Working Paper*.
- Boar, Corina and Virgiliu Midrigan (2021). “Efficient Redistribution.” *Working Paper*.
- Boerma, Job and Ellen R. McGrattan (2020). “Quantifying Efficient Tax Reform.” *Working Paper*.
- Brüggemann, Bettina (2021). “Higher taxes at the top: The role of entrepreneurs.” *American Economic Journal: Macroeconomics* 13.3, pp. 1–36.
- Chang, Yongsung and Yena Park (2021). “Optimal taxation with private insurance.” *The Review of Economic Studies* 88.6, pp. 2766–2798.
- Conesa, Juan Carlos and Dirk Krueger (2006). “On the optimal progressivity of the income tax code.” *Journal of Monetary Economics* 53.7, pp. 1425–1450.
- Conesa, Juan Carlos, Bo Li, and Qian Li (2021). “A Quantitative Evaluation of Universal Basic Income.” *Working Paper*.
- Daruich, Diego and Raquel Fernández (2021). “Universal Basic Income: A Dynamic Assessment.” *Working Paper*.

- De Nardi, Mariacristina, Giulio Fella, and Gonzalo Paz-Pardo (2019). “Nonlinear household earnings dynamics, self-insurance, and welfare.” *Journal of the European Economic Association*.
- Diamond, Peter A. (1998). “Optimal income taxation: an example with a U-shaped pattern of optimal marginal tax rates.” *The American Economic Review*, pp. 83–95.
- Domeij, David and Jonathan Heathcote (2004). “On the distributional effects of reducing capital taxes.” *International Economic Review* 45.2, pp. 523–554.
- Dyrda, Sebastian and Marcelo Pedroni (2021). “Optimal fiscal policy in a model with uninsurable idiosyncratic shocks.” *Working Paper*.
- Farhi, Emmanuel and Iván Werning (2013). “Insurance and taxation over the life cycle.” *The Review of Economic Studies* 80.2, pp. 596–635.
- Farmer, Leland E. and Alexis Akira Toda (2017). “Discretizing nonlinear, non-Gaussian Markov processes with exact conditional moments.” *Quantitative Economics* 8.2, pp. 651–683.
- Feldstein, Martin S. (1969). “The effects of taxation on risk taking.” *Journal of Political Economy* 77.5, pp. 755–764.
- Findeisen, Sebastian and Dominik Sachs (2017). “Redistribution and insurance with simple tax instruments.” *Journal of Public Economics* 146, pp. 58–78.
- Golosov, Mikhail, Maxim Troshkin, and Aleh Tsyvinski (2016). “Redistribution and social insurance.” *The American Economic Review* 106.2, pp. 359–86.
- Golosov, Mikhail, Michael Graber, Magne Mogstad, and David Novgorodsky (2021). “How Americans respond to idiosyncratic and exogenous changes in household wealth and unearned income.” *Working Paper*.
- Greenwood, Jeremy, Zvi Hercowitz, and Gregory W. Huffman (1988). “Investment, capacity utilization, and the real business cycle.” *The American Economic Review*, pp. 402–417.
- Guner, Nezih, Remzi Kaygusuz, and Gustavo Ventura (2021). “Rethinking the Welfare State.” *Working Paper*.
- Guner, Nezih, Martin Lopez-Daneri, and Gustavo Ventura (2016). “Heterogeneity and Government revenues: Higher taxes at the top?” *Journal of Monetary Economics* 80, pp. 69–85.
- Guvenen, Fatih, Fatih Karahan, Serdar Ozkan, and Jae Song (2021). “What Do Data on Millions of US Workers Reveal about Life-Cycle Earnings Risk?” *Econometrica* 89.5, pp. 2303–2339.

- Heathcote, Jonathan, Kjetil Storesletten, and Giovanni L. Violante (2014). “Consumption and labor supply with partial insurance: An analytical framework.” *The American Economic Review* 104.7, pp. 2075–2126.
- (2017). “Optimal tax progressivity: An analytical framework.” *The Quarterly Journal of Economics* 132.4, pp. 1693–1754.
- Heathcote, Jonathan and Hitoshi Tsujiyama (2021). “Optimal income taxation: Mirrlees meets Ramsey.” *Journal of Political Economy* 129.11, pp. 3141–3184.
- Holter, Hans A., Dirk Krueger, and Serhiy Stepanchuk (2019). “How do tax progressivity and household heterogeneity affect Laffer curves?” *Quantitative Economics* 10.4, pp. 1317–1356.
- Hubmer, Joachim, Per Krusell, and Anthony A. Smith (2020). “Sources of US wealth inequality: Past, present, and future.” *NBER Macroeconomics Annual 2020, Volume 35*. University of Chicago Press.
- Jaimovich, Nir, Itay Saporta-Eksten, Ofer Setty, and Yaniv Yedid-Levi (2022). “Universal basic income: Inspecting the mechanisms.” *Working Paper*.
- Kapička, Marek (2013). “Efficient allocations in dynamic private information economies with persistent shocks: A first-order approach.” *Review of Economic Studies* 80.3, pp. 1027–1054.
- Kindermann, Fabian and Dirk Krueger (2021). “High Marginal Tax Rates on the Top 1%? Lessons from a Life Cycle Model with Idiosyncratic Income Risk.” *Forthcoming in the American Economic Journal: Macroeconomics*.
- Krueger, Dirk and Alexander Ludwig (2016). “On the optimal provision of social insurance: Progressive taxation versus education subsidies in general equilibrium.” *Journal of Monetary Economics* 77, pp. 72–98.
- Krusell, Per and Anthony A. Smith (1998). “Income and wealth heterogeneity in the macroeconomy.” *Journal of Political Economy* 106.5, pp. 867–896.
- Lopez-Daneri, Martin (2016). “NIT picking: The macroeconomic effects of a Negative Income Tax.” *Journal of Economic Dynamics and Control* 68, pp. 1–16.
- Luduvic, André Victor Doherty (2021). “The Macroeconomic Effects of Universal Basic Income Programs.” *Working Paper*.
- Mankiw, N. Gregory, Matthew Weinzierl, and Danny Yagan (2009). “Optimal taxation in theory and practice.” *Journal of Economic Perspectives* 23.4, pp. 147–74.
- Mirrlees, James A. (1971). “An exploration in the theory of optimum income taxation.” *The Review of Economic Studies* 38.2, pp. 175–208.

- Panel Study of Income Dynamics (2021). “Produced and distributed by the Survey Research Center, Institute for Social Research, University of Michigan, Ann Arbor, MI (2021).” *Dataset*.
- Park, Yena (2022). “Tax Incidence and Optimal Taxation with General Equilibrium Effects and Transition.” *Working Paper*.
- Peterman, William B. (2016). “The effect of endogenous human capital accumulation on optimal taxation.” *Review of Economic Dynamics* 21, pp. 46–71.
- Piketty, Thomas and Emmanuel Saez (2003). “Income inequality in the United States, 1913–1998.” *The Quarterly Journal of Economics* 118.1, pp. 1–41.
- Rauh, Christopher and Marcelo Rodrigues Santos (2022). “How Do Transfers and Universal Basic Income Impact the Labor Market and Inequality?” *Working Paper*.
- Saez, Emmanuel (2001). “Using elasticities to derive optimal income tax rates.” *The Review of Economic Studies* 68.1, pp. 205–229.
- Stantcheva, Stefanie (2017). “Optimal taxation and human capital policies over the life cycle.” *Journal of Political Economy* 125.6, pp. 1931–1990.
- Trabandt, Mathias and Harald Uhlig (2011). “The Laffer Curve Revisited.” *Journal of Monetary Economics* 58.4, pp. 305–327.
- Young, Eric R. (2010). “Solving the incomplete markets model with aggregate uncertainty using the Krusell–Smith algorithm and non-stochastic simulations.” *Journal of Economic Dynamics and Control* 34.1, pp. 36–41.

## A Analytical model

### A.1 Representative agent

As a special case of the analytical model, we first consider a representative agent. We show that (1) for any level of transfer there exists a progressivity that implements the efficient allocation, and (2) this progressivity is decreasing in the transfer.

*Efficient allocation.*—The first-best allocation maximizes utility subject to the resource constraint (4):

$$\max_{c,n} \log c - B \frac{n^{1+\varphi}}{1+\varphi} \text{ s.t. } c + G = n.$$

The first-order condition of this problem characterizes the efficient labor  $n^*(G)$ :

$$Bn^\varphi(n - G) = 1. \tag{19}$$

*Equilibrium with taxes and transfers.*—Consider a government that has access to a log-linear

income-tax function and a lump-sum transfer. The household chooses  $\{c, n\}$  to maximize utility (1) given the budget constraint (2). The household first-order condition is

$$Bn^\varphi = \frac{\lambda(1-\tau)n^{-\tau}}{\lambda n^{1-\tau} + T}. \quad (20)$$

The government budget constraint is

$$n - \lambda n^{1-\tau} = G + T. \quad (21)$$

*Implementation of the efficient allocation.*—For each transfer  $T$ , a planner can implement the first-best allocation  $n^*(G)$  with an income-tax progressivity equal to

$$\tau(G, T) = -\frac{G + T}{n^*(G) - G - T}. \quad (22)$$

To prove this result, replace (22) in (20) to obtain

$$Bn^\varphi (\lambda n^{1-\tau} + T) = \left( \frac{n}{n - G - T} \right) \lambda n^{-\tau},$$

and use (21) to obtain (19). Finally, it is easy to see from (22) that  $\partial\tau(G, T)/\partial T < 0$ , which proves Claim (1).

## A.2 Heterogeneity and no transfers

In this section we derive the closed-form expression for welfare when the government uses only the log-linear tax function.

*Idiosyncratic productivity.*—Define  $\alpha_{i,t} \equiv \log z_{i,t}$ . One can show that  $\mathbb{E}[\alpha_{i,t}] = -\frac{v_\omega}{2(1-\rho^2)}$  and  $\mathbb{V}[\alpha_{i,t}] = \frac{v_\omega}{1-\rho^2}$ . As  $z_{i,t} = \exp(\alpha_{i,t})$ ,  $\mathbb{E}[z_{i,t}] = 1 \ \forall t$ .

*Household problem.*—The consumers solve a static problem: they maximize utility (1) given the budget constraint (2), with  $T = 0$ . Computing the first-order condition, we get:

$$n_{it} = \left( \frac{1-\tau}{B} \right)^{\frac{1}{1+\varphi}} \equiv n_0(\tau). \quad (23)$$

For the rest of the Appendix, we omit the dependence of  $n_0$  on  $\tau$  to ease notation.

*Computing  $\lambda$ .*—Output is given by

$$Y_t = \int y_{i,t} d_i = \int \exp(\alpha_{i,t}) n_{i,t} d_i = n_0.$$

To compute  $\lambda$ , we also need to compute

$$\tilde{Y}_t \equiv \int y_{i,t}^{1-\tau} d_i = \int [n_0 \exp(\alpha_{i,t})]^{1-\tau} d_i = n_0^{1-\tau} \exp\left(-\tau \frac{(1-\tau)v_\omega}{2(1-\rho^2)}\right).$$

Therefore, using the government's budget constraint (3), we can express  $\lambda$  as

$$\lambda = \frac{Y - G}{\tilde{Y}} = \frac{n_0 - G}{n_0^{1-\tau}} \exp\left(\tau \frac{(1-\tau)v_\omega}{2(1-\rho^2)}\right) \equiv \lambda_0. \quad (24)$$

*Welfare.*—To compute welfare in closed form, we first plug the equilibrium value of consumption and hours worked into the utility function:

$$\begin{aligned} u_{i,t} &= \log \lambda + (1-\tau) \log(\exp(\alpha_{i,t})n_0) - \frac{B}{1+\varphi} n_0^{1+\varphi} \\ &= \log \lambda + \frac{1-\tau}{1+\varphi} \log\left(\frac{1-\tau}{B}\right) + (1-\tau)\alpha_{i,t} - \frac{1-\tau}{1+\varphi}. \end{aligned}$$

Integrating over the distribution of households, we obtain

$$W(\tau) = \int u_{i,t} d_i = \log \lambda + \frac{1-\tau}{1+\varphi} \log\left(\frac{1-\tau}{B}\right) - \frac{(1-\tau)v_\omega}{2(1-\rho^2)} - \frac{1-\tau}{1+\varphi},$$

and, using the closed-form solution (24) for  $\lambda$ , we obtain equation (8) in the main text.

### A.3 Heterogeneity and transfers

We now derive welfare as a function of progressivity  $\tau$  and the transfer  $T$ . The logic of the derivation is the same as in the previous section. However, we cannot express the policy function for labor in closed form, so we linearize around  $T = 0$ .

*Household problem.*—The first-order condition of the household problem reads

$$Bn_{it}^\varphi = \frac{1}{c_{it}} (\exp(\alpha_{it})n_{it})^{-\tau} \exp(\alpha_{it})\lambda(1-\tau);$$

that is, after rearranging,

$$Bn_{it}^{1+\varphi} + \frac{T}{\lambda} Bn_{it}^{\varphi+\tau} \exp(-(1-\tau)\alpha_{it}) - (1-\tau) = 0. \quad (25)$$

Equation (25) defines the function  $G(T, \lambda, n_{it})$  s.t.  $G(T, \lambda, n_{it}) = 0$ . At the optimum, the labor decision is such that, for a given  $T$ ,  $G(T, \lambda, n_{it}(T, \lambda)) = 0$ .

*Linear approximation of labor policy.*—At  $T = 0 \equiv T_0$ , we know that  $n_{it}(T_0, \cdot) = n_0$ . The implicit function theorem holds, and we can compute the slope of  $n_{it}$  in the neighborhood of  $T_0$  as

$$\left. \frac{\partial n_{it}(T, \lambda)}{\partial T} \right|_{(T_0, n_0, \lambda_0)} = - \frac{\left. \frac{\partial G(T, \lambda, n_{it})}{\partial T} \right|_{(T_0, n_0, \lambda_0)}}{\left. \frac{\partial G(T, \lambda, n_{it})}{\partial n_{it}} \right|_{(T_0, n_0, \lambda_0)}}.$$

Let  $\eta \equiv \exp\left((1-\tau) \frac{v_\omega}{1-\rho^2}\right)$ , where we omit the dependence of  $\eta$  on  $\tau$  and  $v_\omega$  to ease notation. We compute the two partial derivatives and obtain a linear approximation around  $(T_0, n_0, \lambda_0)$  of

$n_{it}(T)$  denoted  $\hat{n}_{it}(T)$ :

$$\hat{n}_{it}(T) = n_0 + T \left. \frac{\partial n_{it}(T)}{\partial T} \right|_{(T_0, n_0, \lambda_0)} = n_0 - \frac{T}{1 + \varphi} \frac{n_0}{n_0 - G} \eta^{-\frac{\tau}{2}} \exp(-(1 - \tau)\alpha_{it}). \quad (26)$$

*Computing  $\lambda$ .*—Again, we need to compute  $Y$  and  $\tilde{Y}$ . We start with approximated output:

$$\hat{Y}_t = \int y_{i,t} d_i = \int \exp(\alpha_{i,t}) n_{i,t} d_i = n_0 - \frac{T}{1 + \varphi} \frac{n_0}{n_0 - G} \eta^{-\tau}. \quad (27)$$

To obtain  $\tilde{Y}$ , we first approximate  $\hat{n}^{1-\tau}$ . Using equation (26) we get

$$\hat{n}_{it}^{1-\tau}(T) = \left[ n_0 - \frac{T}{1 + \varphi} \frac{n_0}{n_0 - G} \eta^{-\frac{\tau}{2}} \exp(-(1 - \tau)\alpha_{it}) \right]^{1-\tau},$$

which we can linearize to obtain

$$\hat{n}_{it}^{1-\tau}(T) = n_0^{1-\tau} - \frac{T}{1 + \varphi} (1 - \tau) \frac{n_0^{1-\tau}}{n_0 - G} \eta^{-\frac{\tau}{2}} \exp(-(1 - \tau)\alpha_{it}).$$

It follows that

$$\begin{aligned} \hat{\tilde{Y}}_t &= \int \left[ n_0^{1-\tau} - \frac{T}{1 + \varphi} (1 - \tau) \frac{n_0^{1-\tau}}{n_0 - G} \eta^{-\frac{\tau}{2}} \exp(-(1 - \tau)\alpha_{it}) \right] \exp[(1 - \tau)\alpha_{it}] d_i \\ &= n_0^{1-\tau} \eta^{-\frac{\tau}{2}} \left[ 1 - \frac{T}{1 + \varphi} \frac{1 - \tau}{n_0 - G} \right]. \end{aligned}$$

Using these expressions, we can compute  $\lambda$  using the government budget constraint (3),

$$\hat{\lambda}(T) = \frac{\hat{Y} - G - T}{\hat{\tilde{Y}}} = \frac{n_0 - \frac{T}{1 + \varphi} \frac{n_0}{n_0 - G} \eta^{-\tau} - G - T}{n_0^{1-\tau} \eta^{-\frac{\tau}{2}} \left[ 1 - \frac{T}{1 + \varphi} \frac{1 - \tau}{n_0 - G} \right]},$$

and, taking derivatives and linearizing around  $T = 0$ ,

$$\hat{\lambda}(T) = \lambda_0 + \frac{T}{1 + \varphi} \frac{1}{\eta^{-\frac{\tau}{2}} n_0^{1-\tau}} \left[ -\frac{n_0}{n_0 - G} \eta^{-\tau} - (\varphi + \tau) \right]. \quad (28)$$

*Welfare.*—We approximate utility around  $T = 0$ . The utility of an agent is given by

$$u_{it} = \log \left[ \lambda [\exp(\alpha_{i,t}) n_{it}]^{1-\tau} + T \right] - \frac{B}{1 + \varphi} n_{it}^{1+\varphi},$$

which, using our expressions (28) for  $\hat{\lambda}(T)$  and (26) for  $\hat{n}_{it}(T)$ , can be approximated as

$$\begin{aligned}\hat{u}_{it} &= u_{it,0} + T \left\{ \frac{\lambda'(T) \exp[(1-\tau)\alpha_{it}] n_0^{1-\tau} - \lambda_0 \frac{1-\tau}{1+\varphi} \frac{n_0^{1-\tau}}{n_0-G} \eta^{-\frac{\tau}{2}} + 1}{\lambda_0 \exp[(1-\tau)\alpha_{it}] n_0^{1-\tau}} + \frac{B}{1+\varphi} \frac{n_0^{1+\varphi}}{n_0-G} \eta^{-\frac{\tau}{2}} \exp[-(1-\tau)\alpha_{it}] \right\} \\ &= u_{it,0} + T \left\{ \frac{1}{1+\varphi} \frac{1}{n_0-G} \left[ -\frac{n_0}{n_0-G} \eta^{-\tau} - (\varphi + \tau) \right] - \frac{1-\tau}{1+\varphi} \frac{\eta^{-\frac{\tau}{2}}}{n_0-G} \exp[-(1-\tau)\alpha_{it}] \dots \right. \\ &\quad \left. + \frac{\eta^{-\frac{\tau}{2}}}{n_0-G} \exp[-(1-\tau)\alpha_{it}] + \frac{B}{1+\varphi} \frac{n_0^{1+\varphi}}{n_0-G} \eta^{-\frac{\tau}{2}} \exp[-(1-\tau)\alpha_{it}] \right\}.\end{aligned}$$

Integrating this equation yields  $W(\tau, T) = W(\tau, 0) + \hat{\Omega}(\tau, v_\omega)T$ , with  $\hat{\Omega}(\tau, v_\omega)$  defined as

$$\hat{\Omega}(\tau, v_\omega) \equiv \frac{1}{1+\varphi} \frac{1}{n_0-G} \left[ -\frac{n_0}{n_0-G} + 1 - \tau \right] + \frac{1}{1+\varphi} \frac{n_0}{(n_0-G)^2} [-\eta^{-\tau} + 1] + \frac{1}{n_0-G} [\eta^{1-\tau} - 1], \quad (29)$$

and where, again, we omit the dependence of  $n_0$  on  $\tau$  and of  $\eta$  on  $\tau$  and  $v_\omega$ .

*Welfare decomposition.*—The first term in equation (29), which equates  $\Omega_e^{ra}(\tau)$  defined in equation (10.a), can be rearranged as

$$-\frac{1}{n_0-G} \frac{1}{1+\varphi} \frac{n_0}{n_0-G} + \frac{1-\tau}{n_0} \frac{1}{1+\varphi} \frac{n_0}{n_0-G}.$$

It is equal to the marginal utility of aggregate consumption  $u_c(C_0) = 1/(n_0-G)$ , multiplied by  $\partial \hat{Y}^{ra}(T)/\partial T$ , where  $\hat{Y}^{ra}(T)$  is defined as the representative-agent version of (27), with  $\eta = 1$ , plus the marginal utility of aggregate leisure  $-u_n(n_0) = Bn_0^\varphi = (1-\tau)/n_0$ , multiplied by  $\partial \hat{n}^{ra}(T)/\partial T$ , where  $\hat{n}^{ra}(T)$  is defined as the representative-agent version of (26), with  $\eta = 1$ .

The second term in equation (29) can be rearranged as

$$\frac{1}{n_0-G} \left[ -\frac{1}{1+\varphi} \frac{n_0}{n_0-G} \eta^{-\tau} + \frac{1}{1+\varphi} \frac{n_0}{n_0-G} \right].$$

It is equal to the marginal utility of aggregate consumption  $u_c(C_0)$ , multiplied by  $\partial \hat{Y}(T)/\partial T$  minus  $\partial \hat{Y}^{ra}(T)/\partial T$ . Approximating further using  $\exp(x) \approx 1+x$  delivers  $\Omega_e^{ha}(\tau, v_\omega)$ , defined in (10.b).

The third term in equation (29) can be rewritten as

$$\frac{1}{n_0-G} \eta^{1-\tau} - \frac{1}{n_0-G}$$

and is equal to the average marginal utility across households,  $\int u_c(\lambda_0(z_{it}n_0)^{1-\tau})d_i$ , minus marginal utility of aggregate consumption  $u_c(C_0)$ . Approximating further using  $\exp(x) \approx 1+x$  delivers  $\Omega_r(\tau, v_\omega)$ , defined in equation (10.c).

*Derivatives.*—Claim 2 states that the first term in equation (29),  $\Omega_e^{ra}(\tau)$ , is decreasing in

$\tau$ . To show that, we sign the derivative equal to

$$\frac{1}{1+\varphi} \frac{1}{n_0-G} \left( \frac{n_0}{n_0-G} \frac{1}{1+\varphi} \left[ -\frac{n_0+G}{n_0-G} \frac{1}{1-\tau} + 1 \right] - 1 \right).$$

When  $\tau > 0$ ,  $\frac{1}{1-\tau} > 1$  and  $\frac{n+G}{n-G} \geq 1$  such that  $-\frac{1}{1-\tau} \frac{n+G}{n-G} + 1 < 0$  and the derivative is always negative. More generally, we need to show that the parenthesis is negative. As  $\varphi > 0$ , it is sufficient to show that

$$\frac{n_0}{n_0-G} \left[ -\frac{n_0+G}{n_0-G} \frac{1}{1-\tau} + 1 \right] \leq 1$$

is always true. This condition can be rewritten as

$$-(1-\tau)G^2 \leq n_0(n_0 + \tau G). \quad (30)$$

Equation (30) always holds as the left-hand side is negative  $\forall \tau$ , while the right-hand side is positive as  $\tau \geq -1$  and  $G \leq n_0$  by feasibility.

Claim 3 states that  $\Omega_r(\tau, v_\omega)$ , defined in equation (10.c), is decreasing in  $\tau$ . The derivative reads

$$(1-\tau) \frac{v_\omega}{1-\rho^2} \frac{1}{n_0(\tau)-G} \left( -2 + \frac{1}{1+\varphi} \frac{n_0(\tau)}{n_0(\tau)-G} \right), \quad (31)$$

which is negative  $\forall \tau \in [-1; 1]$  when  $G = 0$ . When  $G > 0$ , equation (31) is negative on  $\tau \in [-1, \hat{\tau}(G)]$ , with  $\hat{\tau}(G) \equiv 1 - BG^{1+\varphi} \left( \frac{2(1+\varphi)}{2(1+\varphi)-1} \right)^{1+\varphi}$ , which is equal to 0.998 in our calibration.

## B Data

### B.1 Taxes and transfers

We use Congressional Budget Office (CBO) data to compute average tax and transfer rates.

We consider only non-elderly households.<sup>27</sup> As an income concept we use a broad measure of market income including wages, employee's contribution for deferred compensation, employer's contribution for health insurance, employer's share of payroll taxes, federal unemployment tax, corporate tax borne by labor, corporate tax borne by capital, capital gains, tax-exempt interest, taxable interest, positive rent, dividends, and other market income.

For taxes, we consider all taxes reported by the CBO: individual income taxes, payroll taxes, corporate income taxes, and excise taxes. Tax credits such as the Earned Income Tax Credit (EITC) and the Child Tax Credit (CTC) are included in taxes in the data. While these credits are at least partially refundable and therefore could be considered transfers, we cannot isolate them in the CBO data and therefore leave them in taxes. State taxes are not reported.

---

<sup>27</sup>As data provided by the CBO is aggregated by quintile over the entire population only, the quintiles do not exactly add up to 20% when excluding elderly households. When comparing tax and transfer rates in the model to the data, we adjust the cutoffs to match the quintile shares in the data: 17.9%, 19.0%, 20.3%, 21.1%, 21.7%.

For transfers, we only consider those that are meant to provide income security. Because our model is an infinite horizon model, we do not consider transfers to the elderly such as social security and Medicare. Also, we do not model health shocks or disability, so we leave Medicaid and Supplemental Security Income (SSI) out of our transfer measure. Our transfer measure includes programs to provide income security such as the Supplemental Nutrition Assistance Program (SNAP; commonly known as food stamps), Temporary Assistance for Needy Families (TANF), housing assistance, and some other smaller means-tested transfer programs.

## B.2 Income and wealth

### B.2.1 Survey of Consumer Finances

The first source for wealth and income data is the 2013 Survey of Consumer Finances (SCF). The unit of observation is a family, defined as the economically dominant single person or couple (whether married or living together as partners) and all other persons in the household who are financially interdependent with that economically dominant person or couple. We restrict the sample to households where the head’s age is between 25 and 60 and only keep observations with labor income (as defined below) above \$5,000. Weights are used throughout. Net worth consists of all real and financial assets net of all debts. Labor income consists of income from wages and salaries. Asset positions are for 2013, whereas income refers to 2012.

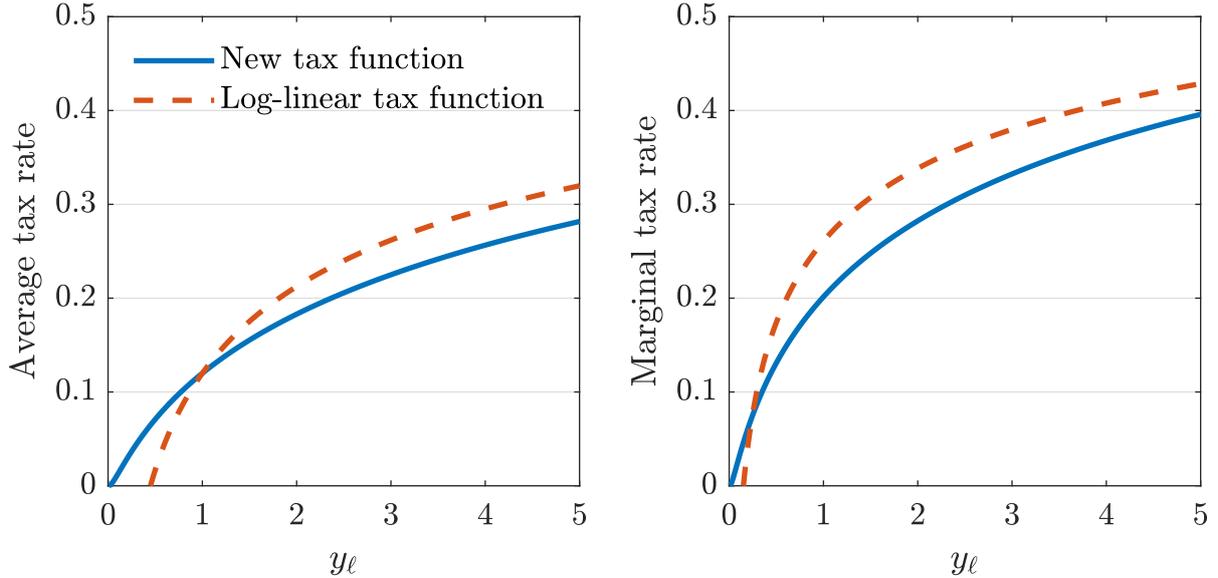
### B.2.2 Panel Study of Income Dynamics

The second data source is the Panel Study of Income Dynamics (PSID), which we use to obtain moments of the earnings growth distribution. We are interested in pre-tax labor earnings at the household level. The PSID was conducted annually between 1968 and 1997 and biannually since. As the model frequency is annual, we restrict ourselves to the surveys before 1997. Furthermore, 1970 earnings questions refer to labor earnings in the previous year, while they refer to the same year before 1970. Hence, we start in 1969 with income information from the 1970 survey. Finally, we stop in 1992 because income definitions change after that point. Labor income includes “labor part of farm income and business income, wages, bonuses, overtime, commissions, professional practice, labor part of income from roomers and boarders”. To obtain household-level income, we add up incomes of the head and wife.<sup>28</sup> We translate incomes to real incomes in 2013 terms using the CPI. Income growth is computed using log differences. We restrict ourselves to households where the head is between ages 25 and 60 and only keep observations with labor income above \$5,000 (in 2013 dollars). We use only the representative Survey Research Center sample.

We compute moments of the earnings growth distribution while keeping the household composition fixed; that is, requiring the head and wife to be the same individuals from  $t - 1$  to  $t + 1$ . We use residualized earnings, taking out year and age effects.

---

<sup>28</sup>PSID terminology for this time period.



**Figure C.1:** New income tax function: comparison to log-linear function

**Notes:** This figure compares average and marginal tax rates implied by the log-linear tax function and the new tax function for  $\tau = \theta = 0.16$ . Labor income is plotted relative to median income.

## C Quantitative model

### C.1 Tax function

Figure C.1 compares average and marginal tax rates implied by the log-linear tax function and the new tax function. In the new tax function, we set  $\theta = 0.16$  and  $\lambda = 0.12$ , as in the calibration. The corresponding parameters in the log-linear tax function are  $\tau = 0.16$  and  $\lambda = 1 - 0.12$ .

### C.2 Model validation: Wealth effects and elasticities

*Wealth effects.*—Golosov et al. (2021) combine data covering the universe of U.S. taxpayers from 1999 to 2016 with data on winnings in state lotteries. 45 U.S. states run a state lottery and winnings above \$600 are reported to tax authorities, as lottery winnings are considered taxable income. They restrict the sample to wins of more than \$30K. They estimate wealth effects on labor earnings for post-win years 1-5 and average across these years.

We replicate this estimation strategy in the model as follows. Starting from the steady state distribution, we increase each household’s assets by the size of the lottery win. Prices and taxes are held constant, as the mass of lottery winners is negligible in the data and winnings are not financed from taxes. We compute average labor earnings of households for the next five years using non-stochastic simulation, similar to Young (2010). As control group, we also simulate a panel of households not experiencing a wealth shock. The difference between the labor earnings of the two groups gives the change in labor earnings due to the wealth shock.

In Table 4 in the main text we report estimates from Golosov et al. (2021) and model implied

**Table C.1:** Wealth effects: model and data

By horizon	Model	Data	Other win sizes	Model	Data
Short run	-2.8	-2.2	Small win	-3.1	-5.6
Long run	-2.4	-2.3	Medium win	-2.0	-2.5

**Notes:** Wealth effects in model and data as reported in [Golosov et al. \(2021\)](#). The model estimates represent the average change in labor income for every \$100 won. The short run response is the average response in years 1-2; the long run response is the response in years 3-5. The small win size is \$50K in the model and between \$30K and \$100K in the data. The medium win size corresponds to \$500K in the model and \$300K-\$1M in the data.

responses for two win sizes. The first size is the average win size of \$180,000. The second size is a large win of \$2,000,000. This corresponds roughly to the present discounted value of a perpetuity of \$25,000, discounted with the calibrated after-tax interest rate of 1.3%.

In [Table C.1](#) we report the effects for two additional win sizes and for different horizons. The model captures that wealth effects are larger for smaller win sizes and relatively stable over the five years after the wealth increase, even though there is some mean reversion in the model.

*Labor elasticity at the top.*—To evaluate labor elasticities in the model, we follow [Kindermann and Krueger \(2021\)](#). In particular, starting from the calibrated steady state, we assume a permanent and unexpected 1%-increase of labor tax rates. The experiment is conducted in partial equilibrium, and without adjusting the government’s budget constraint. We report the labor elasticity of the top-1% income group, using their labor response at the moment of the shock.

## C.3 Numerical solution

### C.3.1 Steady state

To solve for the steady state of the economy, we need to find the real interest rate  $r$  that clears the capital market and the level of transfers  $m$  that clears the government budget constraint. We explain next how we do this.

0. Set grids for assets  $\vec{a}$  and productivity levels  $\vec{z}$ . Let  $N_a = 300$  and  $N_z = 19$  be the number of points in each grid, respectively. Compute the transition matrix of productivities  $\pi_z(z', z)$  using [Farmer and Toda \(2017\)](#).<sup>29</sup>
1. Guess values for the interest rate  $r$  and the transfer parameter  $m$ , and compute the wage  $w$  implied by the guessed  $r$ .
2. Solve for household policies by value function iteration. In particular, for a given guess,

<sup>29</sup>We use the Matlab package provided at <https://sites.google.com/site/aatoda111/discretization>.

guess a value function  $V(a, z)$  and update the value function as

$$\hat{V}(a, z) = \max_{a' \geq 0, n \in [0, 1]} \left\{ \frac{c^{1-\sigma}}{1-\sigma} - B \frac{n^{1+\varphi}}{1+\varphi} + \beta \sum_{z' \in \bar{z}} \pi_z(z', z) V(a', z') \right\}$$

$$\text{s.t. } c + a' \leq wzn + (1+r)a - \mathcal{T}(wzn, ra).$$

Iterate until  $\|\hat{V} - V\| < \varepsilon^V$ . We use  $\varepsilon^V = 1e - 10$ .

3. Compute the stationary measure implied by the optimal policies of step 2. In particular, for a given guess  $\mu(a, z)$ , compute implied measure  $\hat{\mu}(a, z)$  as

$$\hat{\mu}(a_{i'}, z_{j'}) = \sum_{i=1}^{N_a} \sum_{j=1}^{N_z} \mathbb{L}\{a_{i'} = a'(a_i, z_j)\} \pi_z(z_{j'}, z_j) \mu(a_i, z_j)$$

where  $\mathbb{L}$  computes a linear interpolation:  $\mathbb{L}(a_i, a') = \mathbb{I}(a' \in (a_{i-1}, a_i]) \frac{a - a_{i-1}}{a_i - a_{i-1}}$ . Iterate until  $\|\hat{\mu} - \mu\| < \varepsilon^\mu$ . We use  $\varepsilon^\mu = 1e - 11$ .

4. Compute asset market clearing error:  $E^K = A - D - \hat{K}$  where  $A = \int a d\mu(a, z)$  is households' asset holdings and  $\hat{K} = L \left( \frac{r+\delta}{1-\alpha} \right)^{-1/\alpha}$  is capital demand given interest rate and labor supply  $L = \int n(a, z) d\mu(a, z)$ . Also compute government budget constraint error  $E^G = G + rD - \int \mathcal{T}(wzn(a, z), ra) d\mu(a, z)$ . Let  $\mathcal{E}(X) \equiv (E^K, E^G)$  collect the two errors given the guess  $X \equiv \{m, r\}$ . An equilibrium can be written as

$$\mathcal{E}(X) = 0. \tag{32}$$

We solve for  $X$  in equation (32) using a quasi-Newton method.

### C.3.2 One transition

We assume a once-and-for-all fiscal reform, where the two tax parameters  $\lambda$  and  $\theta$  and the transfer phase-out  $\xi$  jump to their new values and the transfer level  $\{m_t\}$  adjusts every period to clear the government budget constraint. We assume that the economy has converged to its new steady state  $\bar{T}$  periods after the shock. We first compute the new steady state as described in Section C.3.1 and obtain the value function  $\bar{V}(a, z)$  and the equilibrium vector  $\bar{X} = (\bar{m}, \bar{r})$  of the new steady state. As the economy has converged in  $\bar{T}$ , we know that the value function at  $t = \bar{T}$  equals its steady-state value  $V_{\bar{T}}(a, z) = \bar{V}(a, z)$ . We also know that the measure at time  $t = 1$  is equal to the initial steady-state value  $\mu_1(a, z) = \mu(a, z)$ . Then, given a guess for transfers and interest rates  $\{m_t, r_t\}_{t=1}^{\bar{T}}$  such that  $(m_{\bar{T}}, r_{\bar{T}}) = (\bar{m}, \bar{r})$ , we solve the household problem backwards and iterate on the sequence  $\{m_t, r_t\}_{t=1}^{\bar{T}}$  using a quasi-Newton algorithm to clear markets.

### C.3.3 Global optimum

To find the optimal tax reform, we rely on the TikTak Global Optimization multistart algorithm—see [Arnoud, Guvenen, and Kleineberg \(2019\)](#) for a detailed description.<sup>30</sup>

We optimize on the triplet  $\{\theta, \lambda, \xi\}$ . For each triplet, we compute the transition as described in Section [C.3.2](#). The algorithm looks for the triplet that maximizes welfare at the implementation of the fiscal reform; that is,

$$\int V_1(a, z) d\mu(a, z).$$

We start local optimizers using the best 30 points in a Sobol sequence of 795 points. The local optimizer is Nelder Mead.

## C.4 Optimal steady state

The fiscal plan that optimizes steady-state welfare is:  $m = 0.37$ ,  $\theta = 0.02$ , and  $\xi = 0$ . In line with [Bakiş, Kaymak, and Poschke \(2015\)](#), the optimal plan is more generous when incorporating transitions: the optimal UBI plan features similar income-tax progressivity but significantly larger transfers, about \$4,000 larger than in steady-state. Furthermore, transfers are optimally a lump sum when optimizing steady-state welfare only, as a phase-out implies a capital tax. When considering only steady states, reducing capital taxation generates large gains, a result that is not robust to transitions—see [Domeij and Heathcote \(2004\)](#) for a detailed discussion of capital taxes and transitions in incomplete-market economies.

We further investigate the optimal UBI plan that optimizes steady-state welfare keeping income-tax progressivity at its status-quo value,  $\theta = 0.16$ . Given the negative relation between  $m$  and  $\theta$ , the optimal level of transfers declines to  $m = 0.27$ , equivalent to \$17,800. This number is larger than in [Guner, Kaygusuz, and Ventura \(2021\)](#), in which the optimal UBI is \$10,400 for a household with two children. Their smaller number may be explained by a combination of factors, such as a richer household structure, lower risk-aversion, and a different calibration of the income process both at the top and at the bottom. In particular, our income process features a Pareto tail and a lower income share of the first income quintile. Both elements increase the optimal transfer for a given level of income-tax progressivity. For instance, when removing the Pareto tail, the optimal transfer declines to \$14,900 when fixing the progressivity to the status quo and optimizing the steady-state welfare.

## C.5 Consumption equivalents and welfare decomposition

### C.5.1 Consumption equivalents

Consumption equivalents  $\gamma(a, z)$  are computed as the increase in consumption in the status quo, keeping status-quo policies fixed, which would make household  $(a, z)$  indifferent between the status quo and the tax reform. More formally, one can write  $V(a, z)$ , the life-time utility of a

---

<sup>30</sup>We use the Fortran package provided at <https://github.com/serdarozkan/TikTak>.

**Table C.2:** Welfare decomposition

Source of welfare gains	Total	Consumption	Leisure
Efficiency	7.28%	-92.00%	99.28%
Redistribution	22.17%	20.32%	1.85%
Insurance	70.55%	76.16%	-5.60%

**Notes:** This table decomposes the welfare gains in three components: efficiency, redistribution, and insurance. Each component is further decomposed in terms of consumption and leisure.

household with state  $(a, z)$  in the status quo, as

$$V(a, z) = \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} - B \frac{n_t^{1+\varphi}}{1+\varphi} \right\} | a, z \right]$$

where expectation  $\mathbb{E}_0$  is taken over future paths of shocks given individual states at the moment of the tax reform.

For a given  $j$ -triplet  $(\theta, \lambda, \xi)$ , let  $V_1^j(a, z)$  be the utility of household  $(a, z)$  when implementing the reform. We compute  $\gamma(a, z)$  such that

$$\mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t \left\{ \frac{[(1 + \gamma(a, z))c_t]^{1-\sigma}}{1-\sigma} - B \frac{n_t^{1+\varphi}}{1+\varphi} \right\} | a, z \right] = V_1^j(a, z).$$

Consumption equivalents are thus equal to

$$\gamma(a, z) = -1 + \left[ 1 + \frac{V_1^j(a, z) - V(a, z)}{\mathcal{C}(a, z)} \right]^{\frac{1}{1-\sigma}}, \quad (33)$$

where  $\mathcal{C}(a, z) = \mathbb{E}_t \left[ \sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} \right\} \right]$  can be computed using a simple iteration of policy functions. Finally, we aggregate over the distribution of consumption equivalents using the measure in the status quo.

### C.5.2 Welfare decomposition

We follow the new decomposition proposed by [Bhandari et al. \(2021\)](#).<sup>31</sup> Table C.2 reports the contribution of aggregate efficiency, redistribution, and insurance to the total welfare gains generated by the tax reform described in Section 4. Each component is further decomposed in terms of leisure and consumption.

<sup>31</sup>We thank the authors for sharing detailed notes on how to implement this decomposition in a related framework.

**Table C.3:** Optimal log-linear plan

<b>Optimal log-linear plan</b>	Q1	Q2	Q3	Q4	Q5
Average $t\&T$ rate	-46%	-12%	5%	17%	46%
Marginal $t\&T$ rate	10%	32%	45%	51%	65%
<b>Optimal plan</b>	Q1	Q2	Q3	Q4	Q5
Average $t\&T$ rate	-155%	-37%	5%	26%	44%
Marginal $t\&T$ rate	61%	66%	62%	54%	51%

**Notes:** This table shows average and marginal  $t\&T$  rates by income quintile for the optimal log-linear plan, and compares them to the optimal plan with the benchmark tax and transfers function.

## C.6 Optimal log-linear plan

We compute the optimal log-linear tax plan using a labor tax function as in the analytical section:  $\mathcal{T}(y) = y - \lambda(y/\bar{y})^{1-\tau}$ . The economy is calibrated in steady state as described in Section 3, and a planner implements a one-time fiscal reform where the benchmark taxes and transfers are replaced by a log-linear function with constant income-tax progressivity  $\tau$ . As in Section 4, we take into account transitions for welfare computations. The optimal plan features  $\tau = 0.49$ . Table C.3 reports average and marginal tax rates.

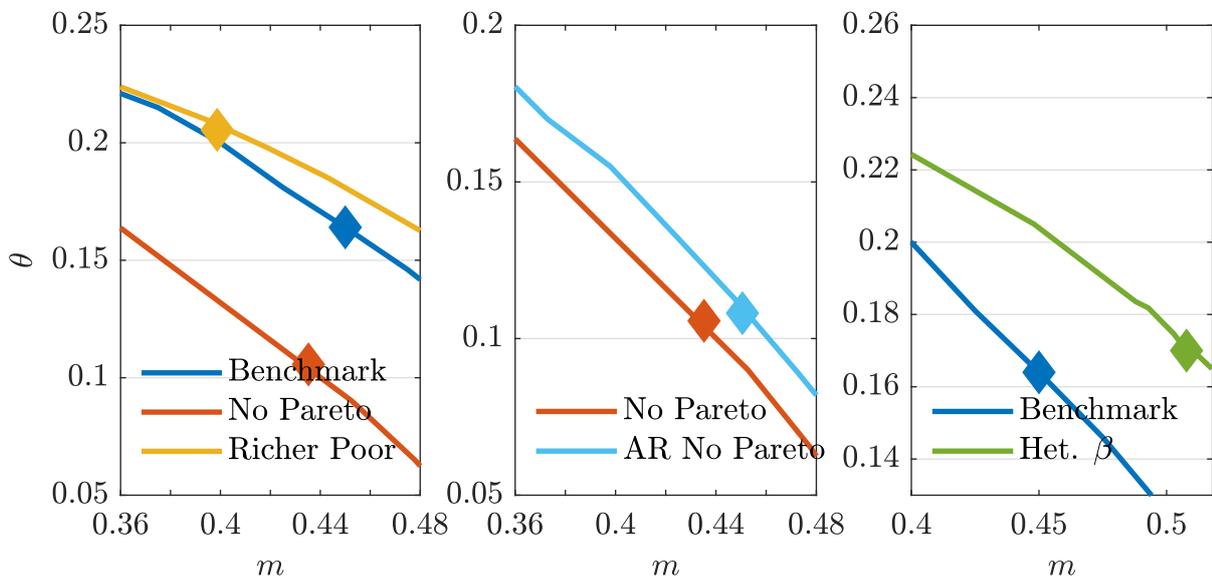
## C.7 The optimal $m$ - $\theta$ line

### C.7.1 The “Heterogeneous $\beta$ ” economy

We calibrate the stochastic process for  $\beta$  as follows.  $\beta$  can take two values:  $\beta_L = 0.943$  and  $\beta_H = 0.994$ . The probability of switching from low to high  $\beta$  is 0.10%, while the probability of switching from high to low  $\beta$  is 0.90%. The stationary distribution features 90% of households with low  $\beta$ . With this stochastic process, the model matches the wealth share of the top 10% (69%, compared to 74% in the data).

### C.7.2 Optimal $m$ - $\theta$ line with positive phase-out

Section 5 plotted the optimal negative relation between transfers and progressivity in the benchmark model, as well as in several alternative calibrations. Throughout that section, we kept the transfer phase-out fixed at  $\xi = 0$ . Figure C.2 plots the optimal  $m$ - $\theta$  line in all economies when the transfer phase-out is at its optimal value,  $\xi = 1.85$ . As expected, all lines shift up, as progressivity is higher when transfers phase-out. Furthermore, all results discussed in Section 5 are robust to using positive  $\xi$ .



**Figure C.2:** Optimal  $m$ - $\theta$  line with positive phase-out

**Notes:** This figure plots the optimal progressivity  $\theta$  for each level of transfer  $m$  in the benchmark, the “No Pareto”, the “Richer Poor”, the “AR No Pareto” and the “Heterogeneous  $\beta$ ” economies. The phase-out parameter is fixed at  $\xi = 1.85$ .