Self-Fulfilling Debt Crises with Long Stagnations*

Joao Ayres
Inter-American Development Bank

Gaston Navarro
Federal Reserve Board

Juan Pablo Nicolini
Federal Reserve Bank of Minneapolis and Universidad Torcuato Di Tella

Pedro Teles
Banco de Portugal, Catolica Lisbon SBE, and CEPR

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Abstract

We assess the quantitative relevance of expectations-driven sovereign debt crises, focusing on the Southern European crisis of the early 2010's and the Argentine default of 2001. The source of multiplicity is the one in Calvo (1988). Key for multiplicity is an output process featuring long periods of either high growth or stagnation that we estimate using data for those countries. We find that expectations-driven debt crises are quantitatively relevant but state dependent, as they only occur during stagnations. Expectations are a major driver explaining default rates and credit spread differences between Spain and Argentina.

Keywords: Self-fulfilling debt crises, sovereign default, multiplicity, stagnations.
JEL codes: E44, F34.

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E-mail addresses: joaoay@iadb.org, gaston.m.navarro@frb.gov, juanpa@minneapolisfed.org, pteles@ucp.pt.

1 Introduction

How important are expectations in triggering sovereign debt crises? In this paper, we explore the quantitative implications of a model of sovereign debt crises that exhibits state-dependent multiplicity. The mechanism we consider to generate multiplicity is the one proposed by Calvo (1988) in which high interest rates induce high default probabilities that in turn justify the high rates. We show that the mechanism is quantitatively relevant, especially during periods of economic stagnation. We build on Ayres et al. (2018), who argue that the mechanism in Calvo (1988) is of interest when the fundamental uncertainty is bimodal, with both good and bad times.[1]

Our analysis of self-fulfilling equilibria in interest rate spreads is motivated by two particular episodes of sovereign debt crises. The first one is the relatively recent European sovereign debt crisis of 2012. The peak of the crisis was in the summer of 2012 and receded substantially after Draghi's "whatever it takes" speech in July, followed by the September policy announcements by the European Central Bank (ECB).[2] The spreads on Italian and Spanish public debt, which were very close to zero since the introduction of the euro and until April 2009, started to increase and exceeded 5% for both Italy and Spain by the time the ECB announced the Outright Monetary Transactions (OMT) program. They were considerably higher in Portugal and, in particular, Ireland, and Greece. With the announcement of the OMT, according to which the central bank stands ready to purchase euro-area sovereign debt in secondary markets, the spreads in most of those countries slid down to less than 2%, even though the ECB did not actually intervene. The potential self-fulfilling nature of the events leading to the high spreads in the summer of 2012 was explicitly used by the president of the ECB to justify the policy.[3]

The second episode is the 1998–2002 Argentine crisis. Back in 1993, Argentina had regained access to international capital markets, but the average country risk spread on dollar-denominated bonds for the period 1993–1999 period, relative to the U.S. bond, was 7%. The debt-to-GDP ratio was only 35% and the average yearly growth rate of GDP was around 5%. Still, the Argentine government defaulted in 2002, after four years of a long recession. Note that a 7% spread on a 35% debt-to-GDP ratio amounts to almost 2.5% of GDP on extra interest payments per year. Accumulated over the 1993–1999 period, this represents 15% of GDP, or almost half of the debt-to-GDP ratio of Argentina in 1993. An obvious question arises: Had Argentina faced lower interest rates, would it have defaulted in 2002?

The main contribution of this paper is to show that the mechanism that generates

[1]See Lorenzoni and Werning (2019) who also discuss multiplicity as in Calvo (1988) using a bimodal endowment distribution.

[2]Draghi's full speech is available at https://www.ecb.europa.eu/press/key/date/2012/html/sp120726.en.html.

[3]See De Grauwe and Ji (2013) on the poor correlation between spreads and fundamentals during the European sovereign debt crisis.

multiplicity in Calvo (1988) is quantitatively relevant. Key for multiplicity is a bimodal output growth process, with persistent good and bad times. We modify an otherwise standard sovereign default model to incorporate an endowment growth rate process that follows a Markov chain, with persistent high- and low-growth regimes. To calibrate the model, we estimate this output process for a set of countries that have recently been exposed to sovereign debt crises. We show that the model features self-fulfilling debt crises, which resemble the two episodes just described.

In particular, the model features equilibria in which interest rates can be high or low, depending on expectations. That is, a sunspot realization can induce discrete jumps in the interest rates faced by the borrower, even with no change in fundamentals. These discrete jumps in rates can only happen if fundamentals are weak. It is only in times of persistently low growth that spreads can be high because of expectations. In the high-growth regime, the region of multiplicity is either empty or negligibly small. Thus, the multiplicity we compute is state dependent: expectations can trigger a crisis only during persistent stagnations.

The schedule of interest rates faced by the borrower can also exhibit discrete jumps because of fundamentals, due to the bimodal growth process. Thus, interest rate jumps are not necessarily a sign of a bad-expectations draw. The interest rate jumps, due to either fundamentals or expectations, induce policy responses by the borrower that can be interpreted as endogenous austerity. The borrower optimally refrains from increasing debt in order to avoid the costs associated with those jumps. Those discrete interest rate jumps can also induce policy responses that resemble gambling for redemption of the type in Conesa and Kehoe (2017), where the borrower raises debt levels beyond the jump in rates. Both endogenous austerity and gambling for redemption are featured in the equilibrium simulations discussed in the paper.

In our quantitative exercises, we consider two calibrations: one that is targeted to Argentina, and one that is targeted to Spain. A key assumption we make is that expectations are more pessimistic for Argentina, meaning that high interest rates are selected more frequently than for Spain. We then explore the effect of assuming either more optimistic or pessimistic expectations for both Argentina and Spain.

We show that the Argentinean calibration generates equilibrium paths that resemble the events of the 2001 debt crisis, with credit spreads jumping by magnitudes similar to the observed ones. While endogenous austerity is present in the calibration for Argentina, the optimal policy eventually switches to gambling for redemption, thus generating high spreads in equilibrium. The calibration for Spain instead only features endogenous austerity. That is, the threat of high spreads triggers endogenous austerity, so that the high spreads are not observed along the equilibrium path. Thus, our model captures the austerity measures implemented by Spain but only in response to off-equilibrium high spreads.

A main finding of our paper is that changes in expectations, as measured by the sunspot probability of selecting an interest rate schedule, have a large quantitative effect on model outcomes. If we assume optimistic expectations for Argentina, default rates and credit spreads decline drastically. That is, without any change in fundamentals, but only in beliefs, Argentina mutates from a serial defaulter to essentially a non-defaulter. Similarly, assuming pessimistic expectations for Spain result in high default rates and credit spreads, even if there is no change in fundamentals. Thus, our model suggests that the difference in expectations is the major driver explaining differences in default rates and credit spreads between Argentina and Spain.

The central results of this paper–that expectations-driven sovereign debt crises are empirically plausible–can contribute to the assessment of the role of policy in sovereign debt crises. It is when fundamentals are weak that a lender of last resort may be called in, not because fundamentals are weak but because the weak fundamentals create conditions for a role of expectations. Of course, the role of the lender of last resort in periods of stagnation will have effects on the economy beyond those periods in which interest rates could be high because of expectations.

Related literature Our model follows the quantitative sovereign debt crises literature that grew out of the work of Eaton and Gersovitz (1981) and was further developed by Aguiar and Gopinath (2006) and Arellano (2008).[4] In these models, a sovereign borrower faces a stochastic endowment and issues non-contingent debt to a large number of risk-neutral lenders. There is no commitment to repay. Timing and choice of actions are important: the assumptions in Aguiar and Gopinath (2006) or Arellano (2008) are that the borrower moves first and chooses the level of non-contingent debt at maturity. We make two main changes to the standard setup. First, we assume that the borrower chooses current debt rather than debt at maturity. This assumption is key to generating multiplicity. When the borrower chooses debt at maturity, it is implicitly choosing the default probability and therefore also the interest rate on the debt. Instead, when the borrower chooses current debt, default may be likely if interest rates are high or unlikely if interest rates are low.[5]

One fragility of the multiplicity mechanism in Calvo (1988) is that, for commonly used distributions of the endowment process, the high-rate schedule is downward sloping, meaning that the interest rates that the country faces decrease with the level of debt. That is not the case if the endowment is drawn from a bimodal distribution with good and bad times, as shown in Ayres et al. (2018) and Lorenzoni and Werning (2019). We depart from the standard setup in assuming that the endowment growth process follows

[4]Other related literature includes Aguiar and Amador (2020), Aguiar et al. (2014), Bocola and Dovis (2019), Cole and Kehoe (2000), Conesa and Kehoe (2017), Corsetti and Dedola (2014), Lorenzoni and Werning (2019), and Roch and Uhlig (2018), among others.

[5]Ayres et al. (2018) show that the timing of moves is also key. In particular, when lenders are first movers, there is multiplicity regardless of whether the borrower chooses current or future debt.

such a bimodal distribution. This change is empirically founded: we estimate a Markov-switching regime for the growth rate of output for Argentina, Brazil, Italy, Portugal, and Spain, and, in all cases, we estimate processes that alternate between persistent high and low growth. We view this feature of the endowment process as reflecting the likelihood of relatively long periods of stagnation in a way that is consistent with the evidence in Kahn and Rich (2007).

The paper closest to ours in its motivation is Lorenzoni and Werning (2019). They also consider the mechanism in Calvo (1988) but exploit a different source of multiplicity due to debt dilution with long maturities. In their environment, multiplicity arises because of the feedback between future and current bond prices: low bond prices tomorrow translate into low bond prices today, while high bond prices tomorrow translate into high prices today. This source of multiplicity is also present in our model but we abstract from it while focusing on multiplicity due to an endowment process with regime switch between persistent high growth and long stagnations. The endowment process is estimated using data of countries exposed to sovereign debt crises.

In the context of self-fulfilling rollover crises as in Cole and Kehoe (2000), Aguiar et al. (2021) also find that simulated moments are sensitive to expectations in a model calibrated to Mexico. We see our papers as complementary, as we explore a different source of multiplicity, the one proposed by Calvo (1988).

Also in the context of self-fulfilling rollover crises, Bocola and Dovis (2019) allow for maturity choice and argue that expectations played a minor role during the European debt crisis. In their environment, governments prefer shorter maturities because of debt dilution incentives, while they prefer longer maturities because of rollover risk. Because maturities were reduced during the European debt crisis, their model suggests that rollover risk was limited during this period. While the source of multiplicity in our model is different, our quantitative exercise also implies a limited role for expectations in the case of Spain: jumps in spreads due to bad expectations do not occur in equilibrium, but only endogenous austerity. There is another important difference in our analysis, since we treat the sunspot probability as a fixed parameter, while Bocola and Dovis (2019) allow for a stochastic process. Our comparative statics exercises show that the probability of the sunspot has a substantial impact on outcomes, thus suggesting that stochastic changes in this probability may account for spreads during the European debt crisis of 2012, an issue that may be worth pursuing in future research.

The paper proceeds as follows. In Section 2, we discuss a simple two-period model to show the key role played by the bimodal distribution in generating multiplicity. In this case, we can derive analytical expressions that highlight the importance of each of the few parameters in the model and provide intuition for the results in the more complex quantitative model of Section 3. In Section 4 we describe the calibration procedure, including the estimation of the endowment process. In Section 5, we discuss the model results and summarize the robustness exercises. Section 6 contains concluding remarks.

2 A two-period model

Here we illustrate the main mechanisms of the model in a simple two-period case. The economy is populated by a representative agent that draws utility from consumption in each period and by a continuum of risk-neutral foreign lenders. The initial wealth of the agent is denoted by ω. The endowment in the second period is distributed according to

$$y_2 = \begin{cases} y^l, & \text{with probability } p \\ y^h, & \text{with probability } (1-p) \end{cases}$$

in which $y^l < y^h$.[6]

The representative agent preferences are given by $u(c_1) + \beta \mathbb{E} u(c_2)$, where u is strictly increasing, strictly concave, and satisfies standard Inada conditions. We assume that the initial wealth and the discount factor β are low enough so that the agent will want to borrow. In period one, the borrower moves first and issues a noncontingent debt level b. Lenders respond with a gross interest rate R. We denote by $R(b)$ the interest rate schedule faced by the borrower. In period two, after observing the endowment y_2, the borrower decides whether to pay the debt or to default. In case of repayment, the borrower consumes the endowment net of debt repayment, $c_2 = y_2 - Rb$. In case of default, there is a penalty expressed as a drop in output to $y^d < y^l$. In addition, the borrower must repay a fraction κ of the debt. Thus, consumption following default is given by $c_2 = y^d - \kappa b$. The agent defaults if the cost of repayment is larger than the benefit:

$$\underbrace{(R-\kappa)\, b}_{\text{cost of repayment}} > \underbrace{y_2 - y^d}_{\text{benefit of repayment}} . \tag{1}$$

In the first period, given initial wealth ω and an interest rate schedule $R(b)$, the borrower solves the following problem:

$$V(\omega) = \max_b \left\{ u(c_1) + \beta \mathbb{E} u(c_2) \right\}, \tag{2}$$

$$\text{subject to} \quad c_1 = \omega + b,$$
$$c_2 = \max \left\{ y_2 - R(b)b, y^d - \kappa b \right\},$$

and is subject to a maximum debt level constraint, $b \leq \overline{B}$.

The assumption that the borrower moves first by choosing a level of debt and that lenders move next with an interest rate schedule is standard. We depart from the lit-

[6] The discrete distribution will help make clear the main mechanisms for multiplicity. We owe this to an insightful discussion by Fernando Alvarez.

erature as in Aguiar and Gopinath (2006) and Arellano (2008) in that we assume that the borrower chooses current debt b rather than debt at maturity, Rb. We follow Calvo (1988) in making this assumption. The risk-neutral lenders will be willing to lend to the agent as long as the expected return is the same as the risk-free rate R^*; that is,

$$R^* = h(R;b) \equiv \left[1 - \Pr\left(y_2 - y^d < (R-\kappa)\, b\right)\right] R + \Pr\left(y_2 - y^d < (R-\kappa)\, b\right) \kappa, \tag{3}$$

in which $h(R;b)$ is the expected return to the lender when the interest rate is R. Given a value for b, the expected return for lenders can be written as

$$h(R;b) = \begin{cases} R, & \text{if } R \leq \frac{y^l - y^d}{b} + \kappa \\ R(1-p) + p\kappa, & \text{if } \frac{y^l - y^d}{b} + \kappa < R \leq \frac{y^h - y^d}{b} + \kappa \\ \kappa, & \text{if } R > \frac{y^h - y^d}{b} + \kappa. \end{cases} \tag{4}$$

In Figure 1, we plot the expected return as a function of the interest rate R, for three levels of debt, together with the risk-free rate R^*. Notice that for low levels of R, the expected return is equal to R since debt is repaid with probability one. In this region, as R increases, the expected return increases one to one. Eventually, R will be high enough that the borrower will default in the low output state, which happens with probability p. At this point, the expected return jumps down. As R increases, the expected return increases at a lower rate, $(1-p)$, since repayment happens only in the high output state. Finally, for high enough R, default will happen with probability one, and the expected return will be the recovery rate κ. A higher level of debt decreases the expected return uniformly, shifting the curves downward.

For low levels of debt, there is only one solution to equation (3), with $R = R^*$. For intermediate levels of debt, there are two solutions: one solution has $R = R^*$ associated with a zero probability of default, and the other has $R = (R^* - p\kappa)/(1-p)$ associated with a probability of default equal to p. For higher levels of debt, the only solution is the high rate $R = (R^* - p\kappa)/(1-p)$. Finally, for even higher debt, there is no solution.

We can now define the following correspondence relating debt levels to interest rates:

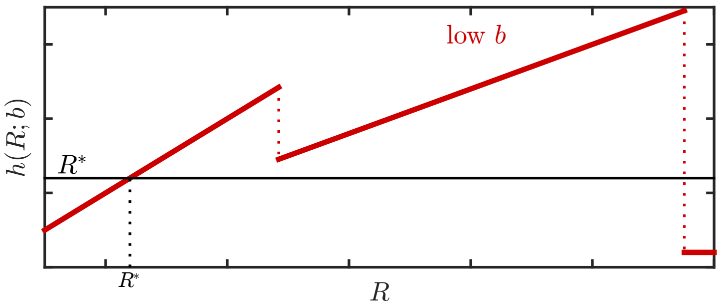
$$\mathcal{R}(b) = \begin{cases} R^*, & \text{if } b \leq \frac{y^l - y^d}{R^* - \kappa} \\ \frac{R^* - p\kappa}{1-p}, & \text{if } (1-p)\frac{y^l - y^d}{R^* - \kappa} < b \leq (1-p)\frac{y^h - y^d}{R^* - \kappa} \\ \infty, & \text{if } b > (1-p)\frac{y^h - y^d}{R^* - \kappa} \end{cases} \tag{5}$$

An *equilibrium* is an interest rate schedule $R(b)$ and a debt policy function $b(\omega)$ such that, given the schedule, the debt policy function solves the problem of the borrower in equation (2), and the schedule $R(b)$ is a selection of the correspondence $\mathcal{R}(b)$.

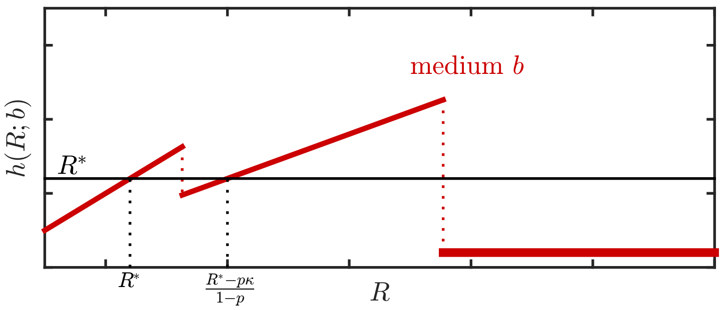
The correspondence $\mathcal{R}(b)$ is plotted (red dashed line) in Figure 2. For all debt levels

Figure 1: Expected return function for different levels of debt

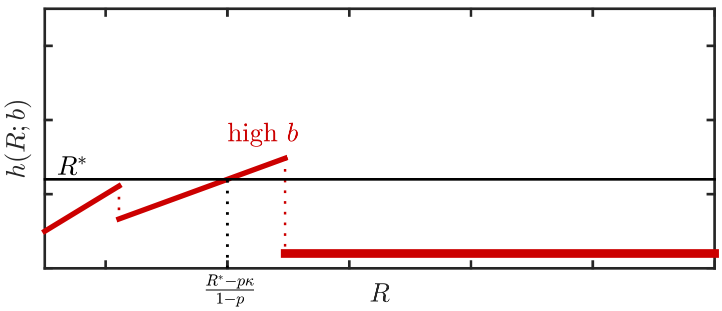
(a) low debt



(b) medium debt



(c) high debt



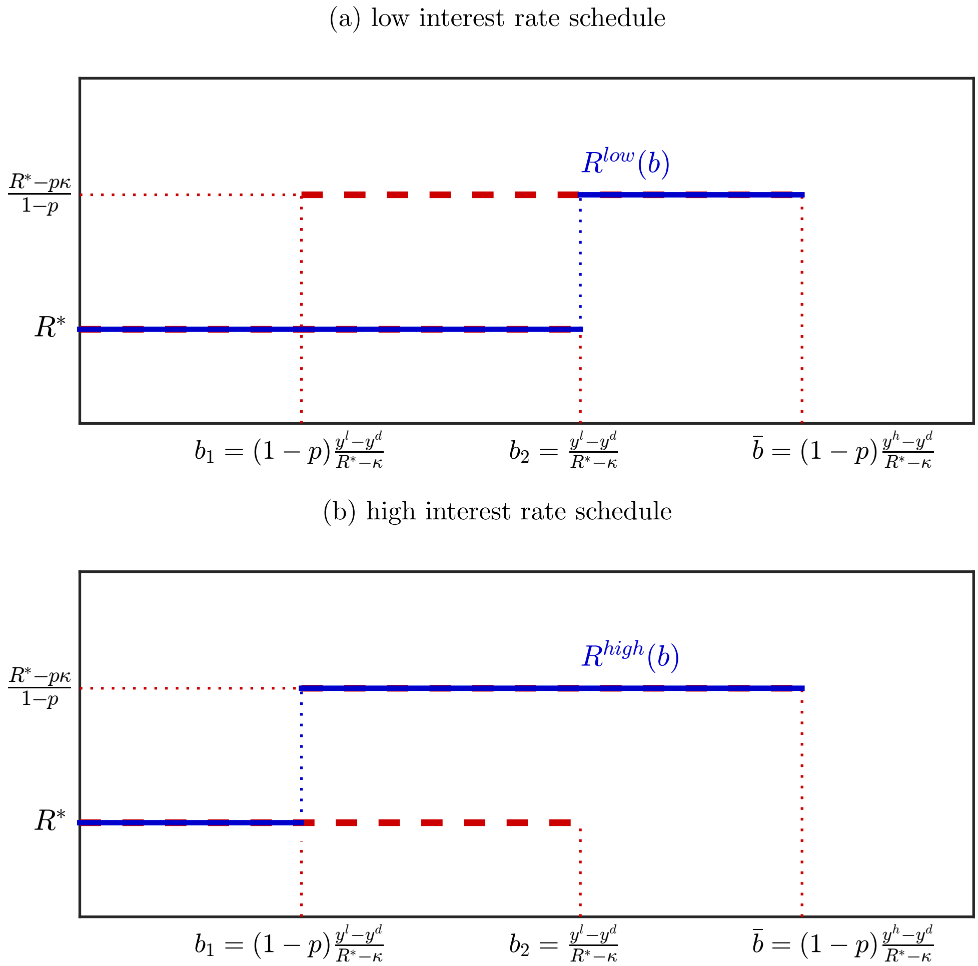
below $b_1 \equiv (1-p)\frac{y^l-y^d}{R^*-\kappa}$, there is only one interest rate, the risk-free rate. For debt levels between b_1 and $b_2 \equiv \frac{y^l-y^d}{R^*-\kappa}$, there are two possible interest rates, the risk-free rate and a high rate. For debt levels between b_2 and $\bar{b} \equiv (1-p)\frac{y^h-y^d}{R^*-\kappa}$, there is again only one interest rate, the high rate. There are multiple interest rate schedules that can be selected from this correspondence. We focus on two of those schedules: a low interest rate schedule, $R^{low}(b)$ in Figure 2a (blue solid line), and a high interest rate schedule, $R^{high}(b)$ in Figure 2b (blue solid line).[7]

We think of b_1 as the debt level above which interest rates jump because of expectations, since alternative expectations could sustain low interest rates. We think of b_2

[7]Notice that it is always the case that $b_1 < b_2$, and that $b_1 < \bar{b}$. However, while Figure 2 represents the case in which $b_2 < \bar{b}$, there are parameter values such that the opposite is true. We chose to plot the case in which $b_2 < \bar{b}$ because this is the case in the quantitative model of Section 3.

as the debt level above which interest rates jump because of fundamentals, since no expectations could sustain lower interest rates. We think of $\bar{b}$ as an endogenous borrowing limit, since any debt issued above this level implies a default probability of one.

Figure 2: Interest rate schedules

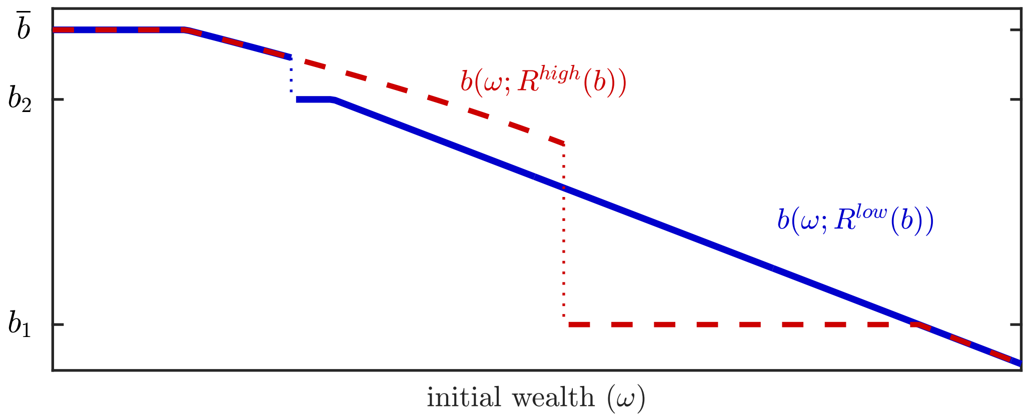


Whether spreads are low or high has implications for the level of debt that can be raised. The region of multiplicity happens for intermediate levels of debt, between b_1 and b_2. If debt is sufficiently low, interest rates can only be low, whereas if debt is sufficiently high, rates can only be high. It is for intermediate levels of debt that interest rates can be either high or low depending on expectations.

Figure 3 shows the optimal debt policy as a function of the initial wealth for the high and low interest rate schedules, for a particular example. For high levels of wealth, the optimal choice of debt is below b_1, and thus the schedule does not matter. As wealth declines, the optimal amount of debt is higher, as indicated by the downward-slopping segment at the bottom right of the figure. Eventually, for low enough wealth, the optimal amount of debt is equal to b_1. At that point, the schedules matter. For the high interest rate schedule, the borrower chooses to keep debt levels at b_1 in order to avoid the discrete jump in interest rates on the whole level of debt. Eventually, for low enough wealth, the marginal utility of consumption in the first period is high enough that the borrower chooses to increase its debt level discretely. This discrete jump shows that the borrower has incentives to avoid at least part of the multiplicity region between b_1 and b_2. As wealth decreases even more, debt levels keep increasing until they reach the endogenous borrowing limit $\bar{b}$. When the borrower faces the low interest rate schedule, borrowing keeps on increasing as wealth declines until it reaches the level b_2. At this point, there is a choice to keep it constant for lower levels of wealth. Eventually, there is also a discrete jump, and debt levels continue to increase until they reach the endogenous borrowing limit.

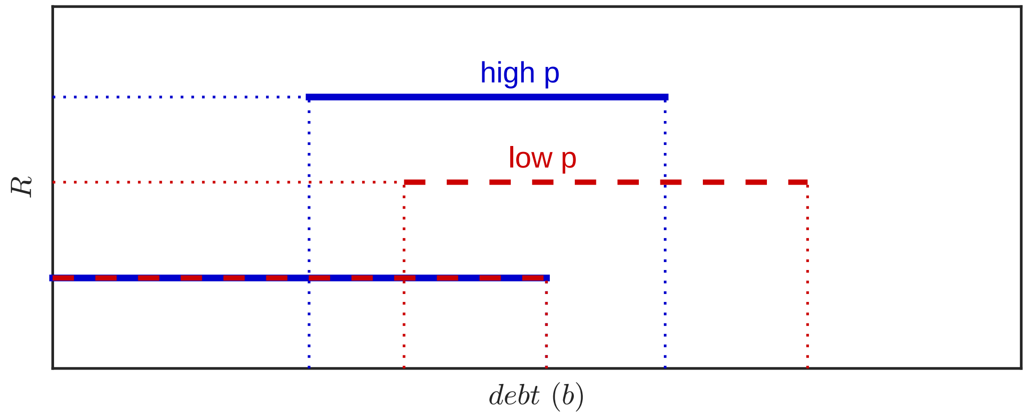
The choice of keeping debt levels constant as wealth decreases happens in our model because of the discrete jumps in interest rates. These jumps can be induced by either expectations, as depicted in the flat region of the red dashed line at the bottom right of Figure 3, or weak fundamentals, as shown in the flat region of the blue solid line at the top left of Figure 3. These flat regions correspond to a form of endogenous austerity, where the borrower adjusts consumption to avoid the high rates. The jumps in the debt policy function indicate decisions by the borrower to end endogenous austerity, increase borrowing discretely, and accept a higher rate. This resembles the gambling for redemption described in Conesa and Kehoe (2017). As we show, the quantitative model of Section 3 exhibits both endogenous austerity and gambling for redemption.

Figure 3: Debt policy function



The probability of the low endowment state We start the comparative statics by considering alternative probabilities of the low endowment state p. Figure 4 plots the interest rate correspondence $\mathcal{R}(b)$ for two values of p. The higher p, the higher is the interest rate that the borrower faces if default happens in the low endowment state. The higher the interest rate, the lower is the minimum debt level such that the borrower defaults in the low state. It follows that a higher p is associated with a higher interest rate and a larger region of multiplicity.

Figure 4: Interest rate correspondence for different p

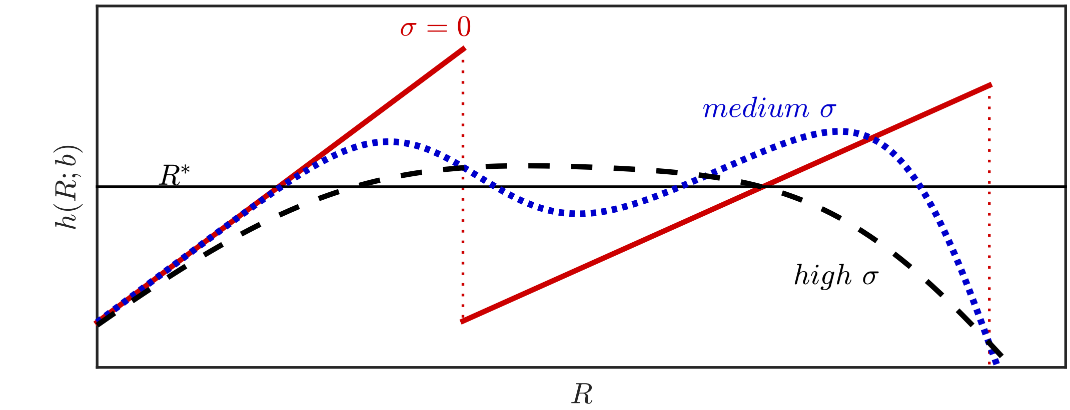


We relate the parameter p to the parameters in the full quantitative model in the next section. There, we have a two-state Markov process in the growth rates of output. The probability of switching to, or remaining at, the low-growth regime is the analog of the value of p in this two-period model. In the estimations described in Section 3, we show that low output growth states are persistent, meaning higher p during stagnations. High output growth states are also very persistent, meaning lower p during expansions. Consequently, in the quantitative model, stagnations come with larger regions of multiplicity and higher interest rates.

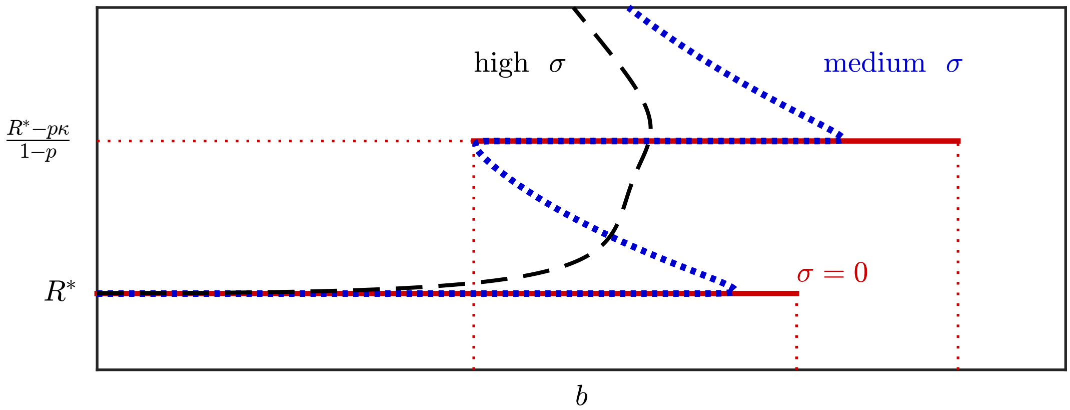
The role of the bimodal distribution In order to highlight the essential role of the bimodal distribution, we consider a generalization in which the endowment in the second period is drawn from a bimodal normal distribution, $y_2 \sim pN(y^l, \sigma^2) + (1 - p)N(y^h, \sigma^2)$. Figure 5 shows, for different values of σ, the expected return function $h(R; b)$ in Figure 5a and the implied interest rate correspondence $\mathcal{R}(b)$ in Figure 5b. The case with $\sigma = 0$ (red solid line) is the one analyzed before, in which there are two solutions to the arbitrage condition in equation (3). For strictly positive small levels of σ (blue dotted line), there are now four solutions to equation (3). However, the two solutions on the downward-sloping part of the expected return function are such that the expected return increases when the interest rate decreases. It follows that those solutions are also on the downward-slopping parts of the interest rates schedules depicted in panel 5b. Furthermore, not only does the interest rate go down with higher current debt b, but total future debt payments, $R(b)b$, also go down when the level of current debt goes up. This also implies that the agent would never choose to be in a decreasing part of the schedule. For these reasons and others, discussed in detail in Ayres et al. (2015) and Lorenzoni and Werning (2019), we rule out solutions along these parts of the interest rate schedule. For higher values of σ (black dashed line), there are two solutions to equation (3), but one can be ruled out. Therefore, to have multiple admissible equilibria, we need to have relatively low levels of σ. We show this is the case in the estimations of Section 3.

Figure 5: Varying the standard deviation of endowment shock, σ

(a) expected return function



(b) interest rate correspondence



The endowment levels y^l and y^h are also important for multiplicity. As y^l approaches y^h, multiplicity disappears as the endowment distribution converges to the unimodal case. A similar rationale will apply to the quantitative results of Section 3.

3 A quantitative model of self-fulfilling debt crises

We calibrate an infinite-horizon model to evaluate the quantitative role of multiplicity in triggering sovereign debt crises. We estimate the endowment process in the model using data on GDP growth for a set of developed as well as developing economies that were exposed to debt crisis episodes. The calibrated model generates self-fulfilling debt crises that can explain the events in Argentina in 2001 and can also shed light on the events in Spain in the 2010s.

3.1 Model

We expand the two-period model of Section 2 to an infinite-horizon framework with output growth and long-term debt. We assume an endowment economy, where output growth follows a two-state Markov process. At the beginning of each period the sovereign chooses whether to default or not on the total stock of outstanding debt. Upon repayment, the sovereign chooses consumption, financed with current endowment and new bond issuance net of repayment. Upon default, the sovereign is excluded from financial markets and loses a fraction of the endowment. While in default, the sovereign may be given the chance to re-enter financial markets. If the decision is to re-enter, the sovereign recovers the totality of the endowment and must honor a fraction of the defaulted debt. Specific details follow below.

Time is discrete, runs forever, and is indexed by $t = 0, 1, 2, \ldots$. We assume a small open economy that receives a stochastic endowment Y_t every period. The preferences of the sovereign are standard,

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma}, \tag{6}$$

where C_t denotes consumption. The utility function is consistent with balanced growth, which allows us to detrend the model as shown below.

Endowment process.—The endowment grows over time as the result of a persistent and a transitory shock. In each period t, the endowment Y_t is given by

$$Y_t = \Gamma_t e^{\sigma \epsilon_t}, \tag{7}$$

$$\Gamma_t = g_t \Gamma_{t-1}, \tag{8}$$

where ϵ_t is *i.i.d.*, $\epsilon_t \sim \mathcal{N}(0, 1)$, and g_t follows a two-state Markov process. Thus, g_t is the current trend growth and Γ_t is the accumulated growth up to period t. We assume that g_t can be either high or low—$g_t \in \{g_H, g_L\}$—representing times of either fast growth or stagnation. The bimodal nature of g_t is empirically plausible and crucial for expectations to play a role in the model.

Debt contract.—The sovereign issues long-term bonds that promise a geometrically decreasing sequence of future payments, governed by the rate $\delta \in [0, 1]$, as in Hatchondo and Martinez (2009). A new debt issuance N_t in period t promises the sequence of payments, starting from $t + 1$,

$$N_t R_t, \quad (1-\delta) N_t R_t, \quad (1-\delta)^2 N_t R_t, \ldots.$$

The value for R_t is determined on the date of issuance, t, and remains constant over the duration of the bond. For $\delta = 1$, R is the gross return on a one-period bond, while for

$\delta = 0$, R is the net interest rate on a consol.

Let B_t denote total payments due at time t because of previous issuance. Thus, we have

$$\begin{aligned} B_t &= N_{t-1}R_{t-1} + (1-\delta)N_{t-2}R_{t-2} + (1-\delta)^2 N_{t-3}R_{t-3} + \dots \\ &= \sum_{j=1}^{\infty} (1-\delta)^{j-1} N_{t-j} R_{t-j}. \end{aligned} \tag{9}$$

This debt contract formulation is convenient because it allows to write debt payments B_t recursively as

$$B_t = (1-\delta)B_{t-1} + R_{t-1}N_{t-1}. \tag{10}$$

We refer to B_t as the debt service in period t.

Let Q be the price of a bond with a payment of $R_t = R$. We compute the interest rate ρ of such a bond so that the present value of promised cash flows equals its price Q. That is, the rate ρ satisfies

$$Q = \sum_{j=1}^{\infty} \frac{(1-\delta)^{j-1}R}{(1+\rho)^j}. \tag{11}$$

Note that the rate ρ is pinned down by the ratio of Q and R. Previous work has typically normalized R and let the price Q be determined upon issuance in equilibrium. Instead, we normalize the price to $Q = 1$ upon issuance and let R be determined in equilibrium. This, together with the assumption that the borrower chooses the current debt issuance, allows for the multiplicity in R as discussed in the two-period model of Section 2. This is a normalization only upon issuance, but the price of the bond can fluctuate thereon, as we discuss below. Then, solving for ρ in (11) yields

$$\rho = R - \delta. \tag{12}$$

While not in default, the resource constraints are

$$C_t + B_t = Y_t + N_t, \tag{13}$$

where, given our normalization, new issuance N_t has a price of one.

We make the same assumptions on the timing of moves and actions of the borrower as in the two-period model. In particular, we assume that the borrower moves first and chooses current debt issuance N_t. Lenders move next and offer a schedule $\mathcal{R}(N_t; B_t, \Gamma_{t-1}, g_t, s_t)$, where s_t is a sunspot variable that selects a particular interest rate when more than one is consistent with an equilibrium.

Sunspot.—The sunspot s_t is the key variable in our model that captures the role of expectations in selecting the equilibrium schedule. In light of the results of Section 2, we allow the sunspot to take two values—$s_t \in \{s_G, s_B\}$—which select a good schedule (low-rate) or bad schedule (high-rate), when more than one schedule is possible. We allow the sunspot to follow a two-state Markov process. However, in our numerical exercises, we will assume that s_t is *i.i.d.* and that the bad sunspot occurs with probability p_B. The value of p_B captures how pessimistic/optimistic the beliefs of lenders are. As we show in the quantitative results of Section 5, the value of p_B has substantial effects on the interest rate schedule as well as behavior of spreads and debt issuance.

Default cost and re-entry.—Default entails two costs for the sovereign. First, the sovereign remains temporarily excluded from financial markets. Second, a fraction of the endowment, $1-\phi_t$, is lost. As is customary in the literature (see Arellano, 2008), we allow for the fraction ϕ_t to depend on the exogenous state $g_t \in \{g_L, g_H\}$. Thus, while in default, the resource constraints are

$$C_t = \phi_t Y_t, \tag{14}$$

with $\phi_t = \phi(g_t)$.

While in default, the option to re-enter credit markets happens with probability $1-\theta$. If the sovereign chooses to re-enter, the output loss, $1-\phi_t$, is lifted and the sovereign regains access to international financial markets. In addition, the service of the debt is resumed, but a fraction $1-\kappa$ of payments is forgone. Thus, lenders recover a fraction κ of the outstanding debt. A sovereign that defaulted in a period with promised debt service B will then face a debt service κB upon re-entry, and future debt payments will evolve as in equation (10).

Values of default and no-default.—Let $V^{nd}(B, \Gamma_-, g, s, \epsilon)$ and $V^d(B, \Gamma_-, g, s, \epsilon)$ be the maximal attainable values of no-default and default, respectively, to a sovereign who starts this period with debt service B, accumulated trend growth Γ_-, current growth g, sunspot s, and output shock ϵ. The value of no-default is

$$\begin{aligned} V^{nd}(B, \Gamma_-, g, s, \epsilon) = \max_{N,C} \Bigg\{ & \frac{C^{1-\gamma}}{1-\gamma} \\ & + \beta \mathbb{E}\left[\max\left\{V^{nd}(B', \Gamma, g', s', \epsilon'), V^d(B', \Gamma, g', s', \epsilon')\right\} | \Gamma_-, g, s\right] \Bigg\} \end{aligned} \tag{15}$$

$$\begin{aligned} s.t. & \\ & C + B = Y + N, \\ & B' = (1-\delta)B + \mathcal{R}(N, B, \Gamma_-, g, s)N, \\ & Y = \Gamma e^{\sigma\epsilon}, \quad \Gamma = g\Gamma_-, \\ & N \leq \bar{N}(B, \Gamma_-, g, s). \end{aligned}$$

The borrowing limit $\bar{N}(\cdot)$ is important in our environment. Since the borrower receives a unit of consumption for every unit of debt issued, default could always be postponed by issuing more debt. This possibility is ruled out by imposing a maximum amount of debt. In practice, we set $\bar{N}(\cdot)$ so that the probability of default next period is never larger than 65%.[8]

The value of default is

$$V^d(B,\Gamma_-,g,s,\epsilon) = \frac{C^{1-\gamma}}{1-\gamma} + \beta\mathbb{E}\Bigg[\theta V^d(B',\Gamma,g',s',\epsilon') \tag{16}$$

$$+ (1-\theta)\max\left\{V^{nd}(\kappa B',\Gamma,g',s',\epsilon'), V^d(B',\Gamma,g',s',\epsilon')\right\}|\Gamma_-,g,s\Bigg]$$

$$s.t.$$

$$C = \phi(g)Y,$$

$$B' = B,$$

$$Y = \Gamma e^{\sigma\epsilon}, \quad \Gamma = g\Gamma_-,$$

where $1-\phi$ is the fraction of output lost upon default. While in default, the service of the debt is suspended. If the sovereign has the possibility to re-enter financial markets and decides to do so, only a fraction κ of the debt service is resumed next period.

Let $\mathbf{C}(B,\Gamma_-,g,s,\epsilon)$ and $\mathbf{N}'(B,\Gamma_-,g,s,\epsilon)$ denote the optimal consumption and debt issuance policies when in no-default, and $\mathbf{B}'(B,\Gamma_-,g,s,\epsilon)$ be the implied debt service next period. Similarly, let $\mathbf{D}(B,\Gamma_-,g,s,\epsilon)$ denote the optimal default policy, and $\mathbf{E}(B,\Gamma_-,g,s,\epsilon)$ denote the optimal re-entry policy while in default. The optimal policies for $\mathbf{D}(\cdot)$ and $\mathbf{E}(\cdot)$ are given by

$$\mathbf{D}(B,\Gamma_-,g,s,\epsilon) = \begin{cases} 0 & \text{if } V^{nd}(B,\Gamma_-,g,s,\epsilon) \geq V^d(B,\Gamma_-,g,s,\epsilon), \\ 1 & \text{otherwise.} \end{cases} \tag{17}$$

$$\mathbf{E}(B,\Gamma_-,g,s,\epsilon) = \begin{cases} 1 & \text{if } V^{nd}(\kappa B,\Gamma_-,g,s,\epsilon) \geq V^d(B,\Gamma_-,g,s,\epsilon), \\ 0 & \text{otherwise.} \end{cases} \tag{18}$$

Pricing of debt.—We assume a continuum of risk-neutral lenders with deep pockets that discount future payments at rate r^*. Let $Z = (\Gamma_-,g,s,\epsilon)$ collect all the exogenous terms in the economy. Let $\mathcal{Q}(B,R,Z)$ be the beginning-of-period (before default decisions) value of a bond that promises R upon issuance, and let $\mathcal{X}(B,R,Z)$ be the value

[8] Similar formulations for a borrowing limit can be found in Chatterjee and Eyigungor (2015) and Hatchondo et al. (2016).

of such a bond in default. Then, $\mathcal{Q}(\cdot)$ and $\mathcal{X}(\cdot)$ are given by

$$\mathcal{Q}(B,R,Z) = [1-\mathbf{D}(B,Z)]\left[R + \frac{1-\delta}{1+r^*}\mathbb{E}\Big[\mathcal{Q}\left(\mathbf{B}'(B,Z),R,Z'\right)|\Gamma_-,g,s\Big]\right] \tag{19}$$
$$+\mathbf{D}(B,Z)\mathcal{X}(B,R,Z),$$
$$\mathcal{X}(B,R,Z) = \frac{1}{1+r^*}\mathbb{E}\Bigg[\theta\mathcal{X}(B,R,Z') + (1-\theta)\left[1-\mathbf{E}(B,Z')\right]\mathcal{X}(B,R,Z') \tag{20}$$
$$+(1-\theta)\mathbf{E}(B,Z')\mathcal{Q}(\kappa B,\kappa R,Z')|\Gamma_-,g,s\Bigg],$$

where equation (20) incorporates that B remains constant during default episodes.

The value of the bond under no default $\mathcal{Q}(\cdot)$ is standard: if the sovereign doesn't default ($\mathbf{D}(\cdot)=0$), R is paid this period and the remaining fraction $1-\delta$ has a next-period value given by $\mathcal{Q}(\cdot)$—evaluated at the next period debt service $\mathbf{B}'(\cdot)$. Notice that, while there is no default, R remains unchanged. The value of the bond in default $\mathcal{X}(\cdot)$ reflects the two possible cases the sovereign will face next period. In the first case, the sovereign remains in default, either because it doesn't have the chance to re-enter financial markets or because it decides not to do so ($\mathbf{E}(\cdot)=0$). In this case, the value of the bond is still given by the function $\mathcal{X}(\cdot)$. In the second case, the sovereign—with probability $1-\theta$—has the chance to re-enter financial markets and decides to do so ($\mathbf{E}(\cdot)=1$). In this case, the lender only recovers a fraction κ of R, but the price of the bond also reflects that the sovereign only has to repay a fraction κ of its former liabilities, which is why the price $\mathcal{Q}(\cdot)$ is evaluated at $\kappa B'$ and κR at the end of equation (20).

Bond price satisfies $\mathcal{Q}(B,R,Z) = R\mathcal{Q}(B,1,Z)$ and $\mathcal{X}(B,R,Z) = R\mathcal{X}(B,1,Z)$.[9] This is an intuitive result: a bond, with an arbitrary payment R, pays, in every state of nature, R times what a bond with a payment of 1 does. Since lenders are risk-neutral, the price of any bond is thus a multiple of the price of a bond with $R=1$. Thus, with a slight abuse of notation, let $\mathcal{Q}(B,Z) = \mathcal{Q}(B,1,Z)$ be the price of a bond with $R=1$. Consequently, the schedule $\mathcal{R}(N,B,\Gamma_-,g,s)$ must satisfy

$$1 = \frac{\mathcal{R}(N,B,\Gamma_-,g,s)}{1+r^*}\mathbb{E}\left[\mathcal{Q}(B',\Gamma,g',s',\epsilon')|\Gamma_-,g,s\right], \tag{21}$$
$$B' = (1-\delta)B + \mathcal{R}(N,B,\Gamma_-,g,s)N. \tag{22}$$

By our normalization, the bond price on issuance—the left-hand-side of equation (21)—is one. Then, for each level of issuance N, $\mathcal{R}(\cdot)$ adjusts endogenously so that the sovereign receives one unit of consumption for each newly issued bond.

[9]Formally, it can be shown that, for any $\lambda > 0$, equations (19) and (20) admit a solution with $\mathcal{Q}(B,\lambda R,Z) = \lambda\mathcal{Q}(B,R,Z)$ and $\mathcal{X}(B,\lambda R,Z) = \lambda\mathcal{X}(B,R,Z)$. The normalization of the price comes from setting $\lambda = 1/R$. See Appendix C for a formal proof.

Equations (21) and (22) are the analog to equation (3) in the simple two-period model, and can be used to discuss the intuition behind the multiplicity results. If lenders coordinate on a high $\mathcal{R}$, this leads—given an issuance N—to a higher B' via equation (22). In turn, a higher B' implies a higher probability of defaulting in the future. That means a lower (expected) $\mathcal{Q}$ next period, which justifies the higher $\mathcal{R}$ in equation (21).[10]

The sunspot captures the role of expectations in selecting the equilibrium interest rate schedule in equations (21)–(22). For the same reasons as discussed in Section 2, the relevant parts of the interest rate schedule are those increasing on issuance. The two increasing interest rate schedules are selected using the same approach as in Section 2. The high-rate schedule corresponds to the highest interest rates for each level of debt, and the low-rate schedule corresponds to the lowest rate.

Equilibrium.—We can now formally define an equilibrium for this economy.

Definition 1 (Equilibrium). *An equilibrium is a set of value functions* $\{V^d(B, \Gamma_-, g, s, \epsilon), V^{nd}(B, \Gamma_-, g, s, \epsilon)\}$, *policy functions* $\mathbf{C}(B, \Gamma_-, g, s, \epsilon)$, $\mathbf{N}(B, \Gamma_-, g, s, \epsilon)$, *default and re-entry functions* $\mathbf{D}(B, \Gamma_-, g, s, \epsilon)$, $\mathbf{E}(B, \Gamma_-, g, s, \epsilon)$, *functions* $\mathcal{Q}(B, \Gamma_-, g, s, \epsilon)$, $\mathcal{X}(B, \Gamma_-, g, s, \epsilon)$ *and* $\mathcal{R}(N, B, \Gamma_-, g, s)$ *such that:*
(i) the policy functions solve the sovereign's problem in equation (15) *and achieve value* $V^{nd}(B, \Gamma_-, g, s, \epsilon)$;
(ii) the value function $V^d(B, \Gamma_-, g, s, \epsilon)$ *satisfies equation* (16);
(iii) default and re-entry policies are as in equations (17) *and* (18);
(iv) functions $\mathcal{Q}(B, \Gamma_-, g, s, \epsilon)$ *and* $\mathcal{X}(B, \Gamma_-, g, s, \epsilon)$ *satisfy equations* (19) *and* (20);
(v) the schedule $\mathcal{R}(N, B, \Gamma_-, g, s)$ *satisfies equation* (21) *with* B' *and* N *as in equation* (22).

3.2 Model normalization

Since the endowment process has a trend, the state variables in the model are non-stationary. For computational purposes, we normalize all non-stationary variables by trend growth Γ_-. This requires showing homogeneity properties of the equilibrium functions, similarly to what was done in Aguiar and Gopinath (2006). We leave the detailed derivation to Appendix C and proceed to present the detrended model.

For any variable X_t, we denote $x_t = X_t/\Gamma_{t-1}$ as the detrended value. Let $v^{nd}(b, g, s, \epsilon)$ and $v^d(b, g, s, \epsilon)$ be the detrended values of no-default and default, respectively. The value

[10] Equations (21) and (22) show that the actions available to the borrower also matter. If the borrower could directly choose next-period debt service B', equation (21) would determine $\mathcal{R}(\cdot)$, and a unique corresponding issuance N would then arise from equation (22). See Ayres et al. (2018) for a detail discussion on how timing and actions lead to multiple equilibria in models of default.

of no default is

$$
\begin{aligned}
v^{nd}(b,g,s,\epsilon) = \max_{c,n} & \left\{ \frac{c^{1-\gamma}}{1-\gamma} + \beta g^{1-\gamma} \mathbb{E}\left[\max\left\{ v^{nd}(b',g',s',\epsilon'), v^{d}(b',g',s',\epsilon') \right\} |g,s \right] \right\}, \\
s.t. & \\
& c + b = y + n, \qquad (23)\\
& gb' = (1-\delta)b + R(n,b,g,s,\epsilon)n, \\
& y = ge^{\sigma\epsilon} \\
& n \leq \bar{n}(b,g,s).
\end{aligned}
$$

where $R(\cdot)$ is the schedule for the detrended variables, as discussed below.

Similarly, the value of default becomes

$$
\begin{aligned}
v^{d}(b,g,s,\epsilon) = & \frac{c^{1-\gamma}}{1-\gamma} + \beta g^{1-\gamma} \mathbb{E}\Bigg[\theta v^{d}(b',g',s',\epsilon') \qquad (24)\\
& + (1-\theta) \max\left\{ v^{nd}(\kappa b',g',s',\epsilon'), v^{d}(b',g',s',\epsilon') \right\} |g,s \Bigg], \\
& c = \phi(g) y, \\
& gb' = b, \\
& y = ge^{\sigma\epsilon}.
\end{aligned}
$$

The schedule offered by foreign lenders is given by

$$
1 = \frac{R(n,b,g,s)}{1+r^{*}} \mathbb{E}\left[Q(b',g',s',\epsilon') |g,s \right], \qquad (25)
$$

$$
gb' = (1-\delta)b + R(n,b,g,s)n, \qquad (26)
$$

where the price $Q(\cdot)$ satisfies

$$
Q(b,z) = [1 - \mathbf{d}(b,z)] \left[1 + \frac{1-\delta}{1+r^{*}} \mathbb{E}\left[Q\left(\mathbf{b}'(b,z), z'\right) |z \right] \right] + \mathbf{d}(b,z) X(b,z), \qquad (27)
$$

$$
X(b,z) = \frac{1}{1+r^{*}} \mathbb{E}\left[\theta X\left(b',z'\right) + (1-\theta) \left\{ \left[1 - \mathbf{e}\left(b',z'\right)\right] X\left(b',z'\right) + \mathbf{e}\left(b',z'\right) \kappa Q\left(\kappa b',z'\right) \right\} |z \right], \qquad (28)
$$

where $z = (g,s,\epsilon)$ collects the stationary exogenous states, $\mathbf{b}'(\cdot)$ in equation (27) is the next period payment implied the optimal issuance policies in equation (23), and where

$b' = b/g$ in equation (28). The default and re-entry policies, $\mathbf{d}(\cdot)$ and $\mathbf{e}(\cdot)$, are given by

$$\mathbf{d}(b, g, s, \epsilon) = \begin{cases} 0 & \text{if } v^{nd}(b, g, s, \epsilon) \geq v^{d}(b, g, s, \epsilon) \\ 1 & \text{otherwise} \end{cases}, \tag{29}$$

$$\mathbf{e}(b, g, s, \epsilon) = \begin{cases} 1 & \text{if } v^{nd}(b, g, s, \epsilon) \geq v^{d}(b, g, s, \epsilon) \\ 0 & \text{otherwise} \end{cases}. \tag{30}$$

4 Calibration

We consider two calibrations of the model, one representing the Southern European experiences, with a focus on Spain, and a second one representing the South American experiences, with a focus on Argentina.

Except for the sunspot probability, we calibrate all parameters using standard values in the literature or matching those parameters directly with data, as we explain in detail below. We cannot do this with the sunspot probability since the parameter is, by definition, arbitrary.

We assume the bad sunspot occurs with probability p_B and use two values for it, one corresponding to pessimistic expectations, and the other to optimistic expectations. We calibrate the case of Spain with a low probability of the bad sunspot, $p_B = 1\%$, reflecting optimistic expectations. The case of Argentina, instead, is calibrated with a high value for the probability of the bad sunspot, $p_B = 25\%$. Given these choices for the sunspot probability, we then calibrate the costs of default using data on total debt and on spreads for each of the two countries, as is customary in the sovereign default literature.

It is worth emphasizing that we explored numerically a wide range of values for p_B between 1% and 50%. The results showed that there is a threshold, around 10% for Argentina and 15% for Spain, such that, for all values below the threshold, the results are very similar to the case of 1%, while for the values above the threshold, the results are very similar to the case of 25%.

We explore the extent to which changing the value of p_B alters the equilibria of the model. In particular, we compute equilibria for both economies assuming the alternative value of the sunspot probability—that is, switching to optimistic expectations for Argentina and to pessimistic expectations for Spain. If expectations did not matter, the outcome should be invariant to p_B. As we show below, the sunspot does matter substantially in both cases.

Common parameters:

A period in the model is one year. We set the annual risk-free rate to $r^* = 3.5\%$. The discount factor is set to $\beta = 0.75$, as is standard in the literature, so that the borrower is impatient enough that borrowing and default are equilibrium outcomes. Arellano (2008) and Chatterjee and Eyigungor (2015) use quarterly discount factors of 0.95 and 0.94, respectively, which imply annual discount factors of 0.82 and 0.77, close to the value we chose.[11] The borrower's risk aversion coefficient is $\gamma = 3$. This value is between the ones used by Arellano (2008), $\gamma = 2$, and Bianchi et al. (2018), $\gamma = 3.3$. Finally, the recovery rate is set to 75% ($\kappa = 0.75$), in line with the estimates in Cruces and Trebesch (2013).[12]

Country-specific parameters:

We use data to discipline the parameters that govern the output process, and the value of δ, which pins down the average maturity of the debt.

For Argentina, we set the average maturity of debt equal to 2.5 years, $\delta = 0.4$, close to the average maturity prior to default in 2001. For Spain, we set the average maturity of debt equal to 6.7 years, $\delta = 0.15$, also close to the average value prior to the debt crisis.

The endowment process in equations (7) and (8) is a regime-switching process characterized by five parameters $\{g_L, g_H, \sigma, p_L, p_H\}$. To calibrate these parameters, we estimate the endowment process using the filter proposed in Kim (1994).[13] We use annual GDP per capita data from The Conference Board Total Economy Database for the period 1980 to 2017 and estimate the process separately for five countries: Argentina, Brazil, Italy, Portugal, and Spain.[14] We start in 1980 to avoid the high growth rates of the period of convergence in the 1960s and 1970s. We assume bounded uniform priors for the five parameters and explore the posterior using a Metropolis-Hastings Markov Chain Monte Carlo (MCMC) algorithm. Table 1 shows the estimates for all countries. As in the model, the estimation assumes the same standard deviation of shocks, σ, across the two growth states. In Appendix A, we also estimate the process allowing for state-dependent σ and find they are similar across states.

Two things are worth mentioning regarding the estimates in Table 1. First, the estimates show clear evidence of a bimodal distribution for output growth in all countries. The average across countries of the difference between g_H and g_L is 6 percentage points, more than three times the standard deviation of the transitory shock. The difference between g_H and g_L is a key ingredient for expectations to play a role.[15] Second, both the low- and high-growth states are persistent (between 60% and 80% persistence). As we show below, this relatively high persistence for the low-growth state generates high

[11] Aguiar and Gopinath (2006) use a quarterly discount factor of 0.8, which corresponds to 0.41 annually. On the other hand, Aguiar et al. (2016) use annual discount factors between 0.84 and 0.89.

[12] The haircut estimates in Cruces and Trebesch (2013) vary from 16% to 40%, corresponding to values of κ between 0.84 and 0.60. We perform sensitivity analysis for different values of κ in the Appendix E.

[13] We do not use the filter in Hamilton (1989) directly because output growth has a moving average component. We use the filter in Kim (1994) instead. See Appendix A for details.

[14] In studying the European experience, we do not include Ireland and Greece in order to concentrate on a relatively homogeneous group of countries.

[15] To reduce the dimensionality of the state, the output innovation in the model is *i.i.d.* One potential concern is that the estimation detects two different regimes as an approximation to a one-regime but persistent process for the growth rate of output. This is not the case: we repeated the estimation assuming an AR(1) process for the innovation and find a statistically and economically significant difference in the growth rates across regimes is estimated also in that case.

Table 1: Prior and posterior distributions

Countries	$\ln(g_L)$	$\ln(g_H)$	p_L	p_H	σ
	Prior distribution				
	$U[-0.1, 0.1]$	$U[-0.1, 0.1]$	$U[0.1, 1.0]$	$U[0.1, 1.0]$	$U[10^{-3}, 0.5]$
	Posterior distribution (mean, and 5th to 95th percentile intervals)				
Italy	-0.017	0.022	0.646	0.843	0.016
	[-0.037,-0.008]	[0.018,0.028]	[0.050,0.990]	[0.627,0.990]	[0.012,0.023]
Portugal	-0.002	0.048	0.805	0.720	0.019
	[-0.011,0.003]	[0.041,0.057]	[0.516,0.990]	[0.454,0.939]	[0.014,0.025]
Spain	-0.018	0.033	0.629	0.838	0.017
	[-0.026,-0.010]	[0.026,0.039]	[0.308,0.990]	[0.653,0.990]	[0.013,0.025]
Argentina	-0.040	0.060	0.620	0.581	0.033
	[-0.049,-0.022]	[0.051,0.078]	[0.346,0.877]	[0.358,0.781]	[0.025,0.044]
Brazil	-0.033	0.029	0.589	0.793	0.019
	[-0.071,-0.022]	[0.025,0.032]	[0.103,0.860]	[0.627,0.923]	[0.014,0.025]

Note: For each country, we estimate an output process as: $\Delta \ln y_t = \ln g_t + \sigma(\epsilon_t - \epsilon_{t-1})$, in which $\epsilon_t \sim N(0,1)$ and $g_t \in \{g_L, g_H\}$, with $\Pr(g_{t+1} = g_L | g_t = g_L) = p_L$ and $\Pr(g_{t+1} = g_H | g_t = g_H) = p_H$. The table reports the mean and the interval between the 5th and 95th percentiles of the posterior distributions of each of the parameters for each country. The table also reports the prior distributions we used, which were chosen to be the same across countries. For each country, we use data on GDP per capita in 2016 US$ (converted to 2016 price level with updated 2011 PPPs) between 1980 and 2017 from The Conference Board Total Economy Database as the measure of y_t. See Appendix A for a description of the estimation.

Table 2: Benchmark calibration

A. Parameters that are common across regions

Description	Parameter	Value
discount factor	β	0.75
risk aversion	γ	3.0
risk-free rate	R^*	1.035
re-entry probability	$1-\theta$	0.10
fraction of debt recovered after default	κ	0.750

B. Parameters that vary across regions

Description	Parameter	Argentina	Spain
inverse of average maturity	δ	0.40	0.15
probability of remaining in low growth	p_L	0.600	0.629
probability of remaining in high growth	p_H	0.750	0.838
low-growth rate	$\ln(g_L)$	-0.033	-0.018
high-growth rate	$\ln(g_H)$	0.029	0.033
standard deviation of transitory shock	σ	0.019	0.017
default cost in high growth state	$\phi(g_H)$	0.90	0.945
default cost in low growth state	$\phi(g_L)$	0.99	0.935
probability of bad sunspot	$p_B = 1 - p_G$	0.25	0.01

but plausible interest rates in the low-growth state.

Table 2 also suggests differences between the Southern European and the South-American countries, the most notable one is that the difference between g_H and g_L is substantially smaller in Europe. Another difference is that the volatility of the disturbance appears to be higher in South America.

When comparing within regions, there is substantial homogeneity in Europe. We therefore chose to use the point estimates for Spain, $p_L = 0.629$, $p_H = 0.838$, $\ln(g_L) = -0.018$, $\ln(g_H) = 0.033$, and $\sigma = 0.015$.

In contrast, in South America there are substantial differences between Brazil and Argentina. Most notably, the difference between g_H and g_L and the volatility of the shock are much higher in Argentina, while the persistence of the high-growth state is much lower. These differences most likely reflect the fact that the data from Argentina incorporate several episodes of default, so the data represents a combination of the true structural parameters, combined with the costs of default, which are calibrated separately.

To be conservative, we use a calibration closer to the estimates for Brazil and set $p_L = 0.60$, $p_H = 0.75$, $\ln(g_L) = -0.33$, $\ln(g_H) = 0.029$, and $\sigma = 0.019$. These values, with the exception of g_H, are within the 90% confidence interval of the posterior distributions of Argentina. The value we pick for g_H is considerably lower than the point estimate for Argentina. We would like to emphasize that choosing a lower value for g_H makes the case for multiplicity harder. Recall that in the simple two-period model discussed above, a higher distance between g_H and g_L increases the region of multiplicity.[16] Table 2 contains the parameter values that we use in the quantitative analysis.

When comparing Argentina to Spain, the two calibrations capture the fact that South American countries have lower average debt maturity, deeper recessions, less persistent periods of high growth, and more volatile output growth and fluctuations overall.

We assume the sunspot follows an *i.i.d* process and denote p_B the probability of the bad sunspot. As discussed before, the calibration for Argentina uses $p_B = 25\%$ while the calibration for Spain uses $p_B = 1\%$.

Finally, we use data on debt levels and spreads over the risk-free rate to calibrate the costs of default for each country. As is standard in the quantitative sovereign default literature, following Arellano (2008), default costs are asymmetric. We choose the values so that the average stock of debt in the simulations are close to their debt position the years before entering the crisis period, and the spreads are close to the ones observed before the economy transitions from high to low growth.[17]

Regarding debt levels, for Spain, we use the measure of net international investment position from Banco de España, which averaged 92% between 2008 and 2012. For Ar-

[16] Appendix E shows comparative statics with respect to the distance between g_H and g_L.

[17] We match average debt levels in the high-growth state, since we want to think of debt crisis periods as being triggered by transitions from high- to low-growth states.

gentina, most foreign assets were reserves held at the central bank, which were earmarked to back the monetary base at the time, due to the currency board. Therefore, we decided not to subtract these reserves from the gross measure of the debt. We use Argentina's gross external debt from the World Bank's International Debt Statistics, which averaged 52% between 1997 and 2001. See Appendix B for details on data sources and computations.

As pointed out by Dias et al. (2014), existing measures of debt for Argentina and Spain are not comparable. The key difference is that while the data for Spain are calculated using market prices, the data for Argentina corresponds to face values of the outstanding bonds. To go around these discrepancies between countries, we measure both the market value and the face value of debt in the model. The market value is Qb/δ, with Q defined in equation (27). The face value is the undiscounted sum of principal payments due in the future, $F_t \equiv N_{t-1} + (1-\delta)N_{t-2} + (1-\delta)^2 N_{t-3} + ...$ We denote its detrended value by f_t. We then use the market value in the model to calibrate the case of Spain and the face value in the model to calibrate the case of Argentina.

Finally, we target about 0% spreads for Spain and 8 percent for Argentina, which correspond to the averages for the years before their respective crises.[18] These are the spread numbers we obtain in the high-growth schedules, as we report in Figures 6 and 8 below. The calibrated default costs are reported in Table 2.

In Section 5, as well as in the Appendix E, we perform sensitivity analysis for many of the parameters described above, and show that the conclusions are reasonably robust to alternative parameter values.

Model computation The computation of the model has to take into account that there are multiple interest rate schedules satisfying equations (25) and (26). That includes schedules that are decreasing in the level of debt, which we rule out (Ayres et al., 2015). That is, the model computation must select schedules out of potentially several possible equilibrium schedules. The infinite-horizon nature of the model introduces an additional complexity because the selection of schedules affects value functions which in turn affects the schedules. To overcome this issue, we develop an algorithm that iterates only on the value function and that, in each iteration, computes the respective interest rate correspondence, selecting the high and low interest rate schedules as a function of the sunspot. Appendix D contains more details on the algorithm used to compute the equilibrium.

[18]For Spain, spreads were essentially zero most years before the Lehman crisis. In contrast, spreads for Argentina varied substantially during 1998, ranging from 4% to 12% and hovering around 8% during the last months of the year.

5 Quantitative results

We present two main results. We first show that for both calibrations there are multiple interest rate schedules for the values for the debt service that Argentina faced in 2001 and Spain faced in 2012. This suggests that expectations may have played a role in triggering debts crises in both economies. The second set of results uses the simulation of the calibrated economies. We assess the relevance of the sunspot for the moments of the simulated economies, in particular for spreads, default rates, and debt levels.

The model calibration for Argentina generates equilibrium paths that resemble the events the country experienced in 2001. The sunspot probability plays a crucial role in generating those paths. In contrast, while the Spain calibration exhibits multiplicity for its 2012 debt service levels, the sunspot induces a strong endogenous austerity response, such that expectations-driven high spreads are not observed in equilibrium.

Recall that, as a benchmark, we set $p_B = 1\%$ for Spain, reflecting optimistic expectations, while we set $p_B = 25\%$ for Argentina, reflecting pessimistic expectations. Beyond expectations, there are three differences between the Argentina and Spain calibrations, all of them disciplined by data: the endowment process, the default costs, and the maturity of the debt. As it turns out, the difference in expectations is the major driver explaining differences in default probabilities and spreads between Spain and Argentina.

In what follows, we start by discussing the schedules for the two calibrated economies and then proceed to discuss both simulated economies.

5.1 Multiplicity of interest rate schedules: Argentina

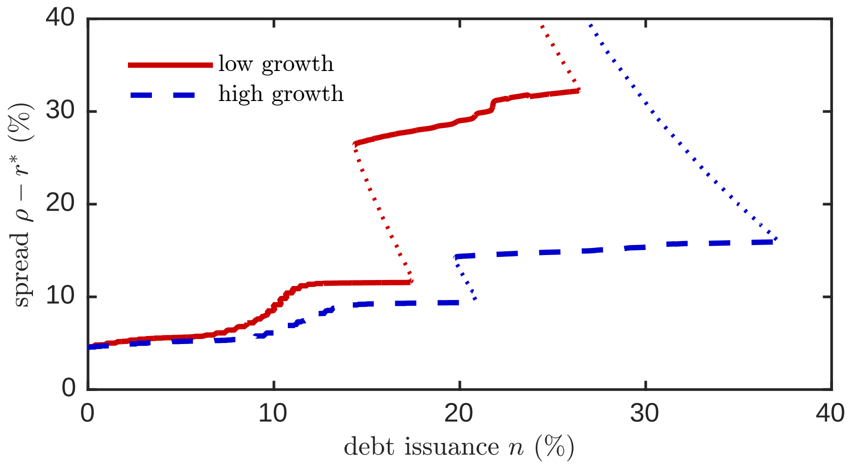
Here we explore the calibration for Argentina, focusing on the sovereign debt crisis of 2001. By January 2001, sovereign debt spreads in Argentina were roughly 8%. From 1994 to 1998, the Argentine economy grew at high rates, close to 6% a year. By the end of 1998, a recession started, lasting until 2002. This can be interpreted as a regime switch from the high- to the low-growth state. Argentina defaulted on its debt in December 2001, after a couple of months with spreads that averaged around 30%.

In Figure 6, we plot the interest rate schedules for the low- and high-growth states, for debt service levels of 20% of GDP, close to what Argentina had in 2001 at the onset of the crisis.[19] The horizontal axis has the new debt issuance, n, while the vertical axis has the corresponding interest rate spreads.[20] For each growth state, the sunspot realization determines whether the high or low interest rate schedule is selected, in the region of multiplicity. The (blue) dashed line corresponds to the high-growth state, while the (red) solid line corresponds to the low-growth state. As discussed above, the Argentina

[19]Debt services in 2000 for Argentina equaled 19.6% of GDP, based on World Bank's International Debt Statistics. See Appendix B for more details on data sources and computations.

[20]Interest spreads are $\rho - r^*$, where ρ is the interest rate on new issuance as defined in equation (12).

Figure 6: Interest rate spreads for Argentina ($b = 20\%$)



calibration was chosen to match an 8% spread in the high-growth schedule, similar to what the country faced during the high-growth years before the crisis.

The size of the multiplicity region is state-dependent, as Figure 6 shows. In the high-growth state (dashed blue line), the region of multiplicity is small, so that the schedules are similar under the good and bad sunspot. Instead, in the low-growth state (solid red line), interest rates can be either low or high, depending on the sunspot, for a larger set of issuance levels. In the low-growth state, there is multiplicity of schedules for issuance between 14% and 18% of output. For these levels of debt issuance, the low spread is around 12%. But there is also a high spread close to 30%. When expectations are good, the borrower can issue up to 18% of trend GDP under the lower 12% spread. Borrowing only 18% when the debt service is 20%, as in Figure 6, implies a surplus of 2%. On the other hand, the country can only issue up to 14% with low spreads if expectations are bad. This means that a surplus of 6% of GDP is needed under the bad sunspot in order to maintain the lower spreads. Thus, under bad expectations, a much larger surplus adjustment is needed in order to obtain a low spread.

A noticeable feature of Figure 6 is that the high spreads resemble the ones for Argentina around the 2001 debt crisis. Spreads in Argentina went up to close to 30% during the last two months of 2001. This number was not targeted in our calibration.

In this model, expectations play a significant role when fundamentals—the growth rate of the economy—are weak. The reason for this state-dependent role of expectations is the following. In periods of low growth, the probability of observing low growth in the future is high, due to the high persistence of the growth shock. Thus, if the borrower is expected to default in the low-growth state, which has a sizable probability, the interest rate must be high. This high interest rate will, in turn, induce the borrower to default in the low-growth state, confirming the expectations. This will happen even for low debt levels. If the borrower is not expected to default in the low-growth state, however, the interest rates will be relatively low, and the borrower will be able to issue a larger amount of debt without risking default next period. This translates into a large region of multiplicity, in which for intermediate levels of debt, interest rates can be either high or low depending on expectations.

In contrast, persistence implies that in periods of high growth, the probability of switching to low growth is low. Thus, even if the borrower is expected to default in the low-growth state, the interest rate consistent with these expectations will be low, since the probability of the low-growth state is low. Expectations will be confirmed, meaning that the borrower will default in the low-growth state, but only when debt levels are relatively large. If the borrower is not expected to default in the low-growth state, however, the interest rates are only marginally lower, and the debt levels such that the borrower will not default are not much higher. This translates into a small region of multiplicity.

The probability of switching to the low-growth state in the infinite-horizon model is the analog of the probability of the low endowment in the two-period model of Section 2. When that probability is low, the region of multiplicity is small, whereas when that probability is high, the region of multiplicity is large. In the infinite-horizon model, the probabilities are functions of the state. In low-growth states, the probability of future low growth is high, and the region of multiplicity is large. In high-growth states, that probability is low, and the region of multiplicity is small.

In what follows, we show how the schedules change as we vary key parameters of the model. We restrict the analysis to the schedules for interest rate spreads in the low-growth state, since that is when multiplicity is more prevalent, as discussed above. Given the focus of our analysis, we are particularly interested in the effects of changing the probability of the sunspot, the debt service, and the average maturity. A full set of robustness results is presented in Appendix E.

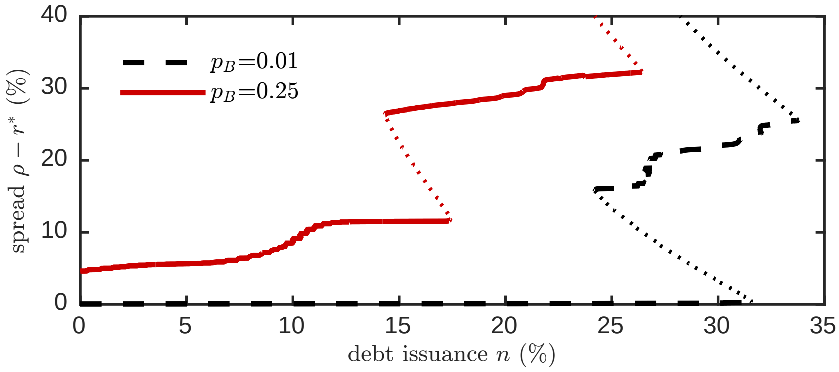
The role of the sunspot We compute the schedule setting the probability of the bad sunspot to $p_B = 1\%$, rather than $p_B = 25\%$ as used for the benchmark.[21] The schedule for this case, when the growth rate is low, is depicted in Figure 7a (black dashed line) together with the one corresponding to the benchmark (red solid line).

The effect of the change in p_B is striking. If the probability of the bad sunspot was very low, the country would have faced zero spreads even if it were to issue new debt up to almost 25% of GDP. In our narrative, this implies that, with optimistic expectations, Argentina could have been able to borrow to service the debt plus a few extra points of GDP and still be well within the debt choices that essentially rule out default, which explains the zero spread.

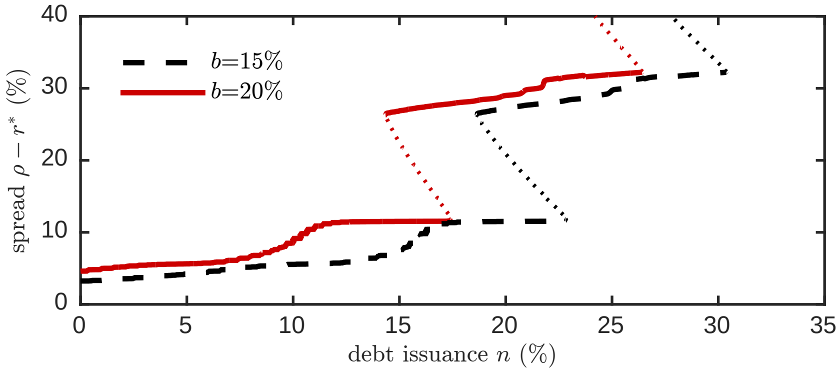
[21] As mentioned above, we have solved the model for several values of the probability of the bad sunspot, p_B, between 0% and 50%. The solution essentially depends on its value being above or below a threshold close to 10%. For values below the threshold, the results are very similar to the case of $p_B = 1\%$, while for values above the threshold, the results are very similar to $p_B = 25\%$.

Figure 7: Interest rate spreads for Argentina in the low growth state ($b = 20\%$): Comparative statics

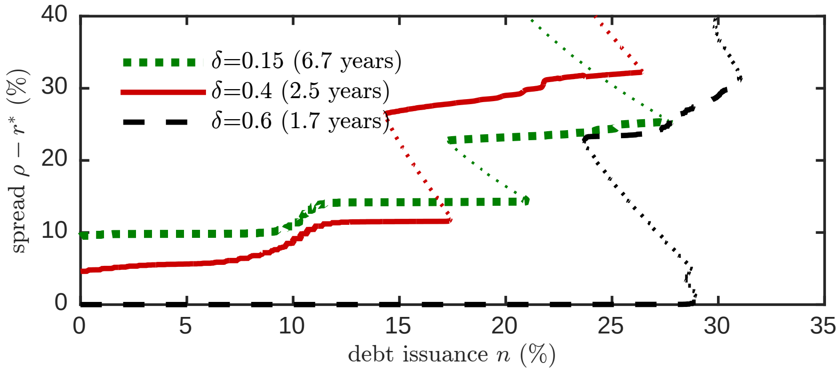
(a) The role of expectation p_B



(b) Inherited debt b



(c) Maturity δ ($b/\delta = 50\%$)



The role of the inherited debt We now show the effects of the inherited debt service b in Figure 7b. The solid red line is the same one depicted in Figure 6, corresponding to the benchmark. The black dashed line corresponds to a debt service of $b = 15\%$. As the figure shows, it is possible to roll over a debt service of 15% with a single low interest rate. Only if the government attempts to borrow another 4% of output, for a total of 19%, will multiplicity potentially matter.

The role of average maturity To assess the role of the average maturity δ, Figure 7c reproduces the schedule for the benchmark calibration, together with the ones for either higher or lower average maturity. As before, we only show the schedules for the low-growth state where multiplicity is quantitatively relevant. We change the average maturity but keep total debt constant, so that the service of the debt, b, changes accordingly. Together with the case of the benchmark average maturity of two and a half years, plotted in Figure 6, we also plot the cases of maturity of 6.7 years ($\delta = 0.15$) and 1.7 years ($\delta = 0.6$).

The maturity of debt presents a tradeoff. As the maturity increases, the incentive to dilute the debt is higher, which pushes spreads up. On the other hand, given a total level of debt, the current debt services are lower the longer the maturity. Thus, as Figure 7c shows, the schedule increases with higher maturity, but as the debt services are lower, for a given deficit, there is a movement within the schedule to the left, away from the multiplicity region. For example, with a 50% debt-to-output ratio and $\delta = 0.15$, only 7.5% of output needs to be serviced every period. That value is comfortably below the region of multiplicity. Thus, while the interest rate is higher for any given level of debt, according to the model a higher maturity could have kept Argentina away from a crisis, even with positive but mild deficits.

The case of a shorter maturity, of 1.7 years ($\delta = 0.60$), is quite different. The spreads are much lower than in our benchmark. It is so much so that the spread is zero, even for new debt issuance as large as 30% of output. This is quite remarkable, since it implies that longer maturities do not necessarily translate into better financing conditions. The interpretation is that, for the short maturity, debt dilution incentives are minor.[22] This effect, combined with a level of debt that is not very high, implies that there is almost no default in equilibrium. This example highlights the interaction between incentives to default down the road and the maturity structure. A longer maturity is good, since it does not require a high fraction of the debt to be rolled over in a single period. On the other hand, it makes the debt dilution problem more acute. And the two effects feed back into each other: as the average maturity goes up, the debt dilution problem implies higher rates, which in turn imply a higher burden for the debt and higher incentives to

[22] See Chatterjee and Eyigungor (2012) and Aguiar et al. (2019).

default. And this higher probability of default makes the debt dilution more severe.[23] For the short average maturity we are considering, the reversion of this feedback is so strong that default becomes an almost impossible event, making the debt dilution component of the spread vanish. One could be tempted to conclude that short maturity debt has attractive features from a policy standpoint. However, the size of the implied debt dilution spread in this family of models is counterfactual. Thus, we believe these implications ought to be taken with caution. For a discussion on this issue, see Aguiar and Amador (2020) and chapter 7 in Aguiar and Amador (2021).

5.2 Multiplicity of interest rate schedules: Spain

We now explore the calibration for Spain. Before the crisis, Spain was in the high-growth regime and the spread was essentially zero. Following the 2009-10 recession, growth was dismal, which we interpret as the-low growth regime. Spreads went up steadily to levels higher than 6% by July 2012, when the ECB announced a policy intervention. Spreads then fell consistently to reach 1% by 2014.

In Figure 8, we plot the interest rate schedules for both the low- and high-growth states. In contrast to Argentina, we don't have direct data on Spain's debt services due each year. However, we can impute debt services using data on short and long debt, interest payments, and the maturity structure.[24] For 2011 we obtain an average debt service relative to GDP of $b = 15\%$, the value we use to plot Spain's schedules. As discussed above, the Spain calibration was chosen to match a spread of essentially zero in the schedule corresponding to the high growth state.

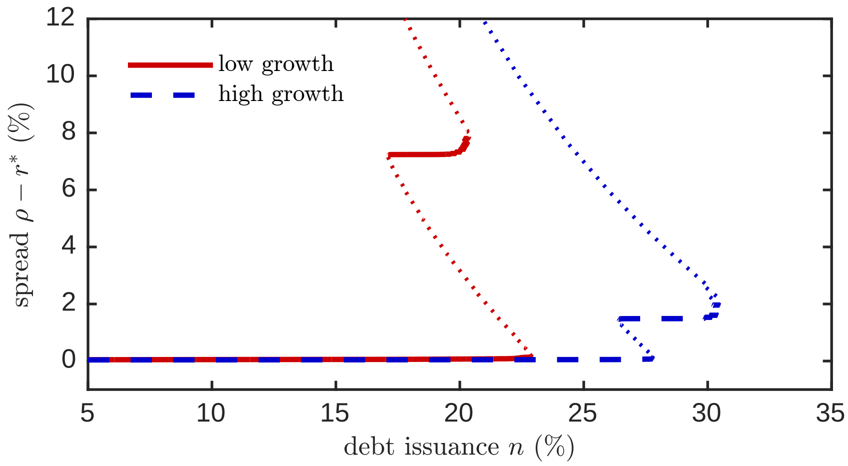
For the low-growth state, for debt issuance levels between 17% and 22% of output, interest rates can be either low or high depending on the sunspot, as Figure 8 shows. When expectations are good, the borrower can issue new debt up to 23% of trend GDP under the lower interest rate spread. In contrast, when expectations are bad, the borrower can only issue up to 17% of trend GDP under the lower spreads. This means that, given a debt service of 15%, there is multiplicity of spreads for deficits of 2% to 8%. The deficits Spain experienced in the years before the debt crisis belong to those intervals.

Figure 8 supports a narrative regarding the debt crisis in Spain consistent with multiple equilibria. By 2009, Spain had entered the low-growth regime, so the relevant schedule is the red one in Figure 8. While expectations were good, Spain could run deficits, with a low spread. Eventually, the bad sunspot realizes and spreads jump up. As Figure 8 shows, the difference between the high and low interest rate spreads in the multiplicity region—which has not been calibrated—is about 7%. This difference in spreads is very similar to the maximum spread observed in Spain (slightly above 6%).

[23]This feedback has been shown to generate multiple equilibria by Lorenzoni and Werning (2019) and Aguiar and Amador (2020).

[24]See Appendix B for details.

Figure 8: Interest rate correspondence for Southern Europe ($b = 15\%$)



The narrative for Spain is partially confirmed by the analysis of the simulated economy, as we discuss below. Endogenous austerity plays a key role, avoiding the high spreads, so that the economy does not borrow at expectations-driven high rates in equilibrium.

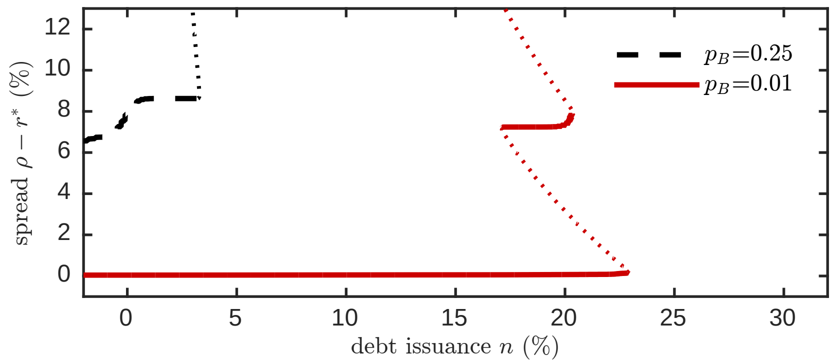
The role of the sunspot The benchmark for Spain assumes a low sunspot probability, at $p_B = 1\%$. We now explore the effect of increasing the sunspot probability to $p_B = 25\%$, as in the benchmark for Argentina. Figure 9a plots the schedule under the low-growth state for both $p_B = 1\%$ and $p_B = 25\%$.

As with Argentina, the value of p_B substantially changes the interest rate schedules. With more pessimistic expectations, spreads increase sharply and Spain's capacity to borrow is substantially reduced. We show this change in schedules carries its effect on equilibrium outcomes: had Spain faced the same value of p_B as Argentina, spreads and default rates would have been close to Argentina's.

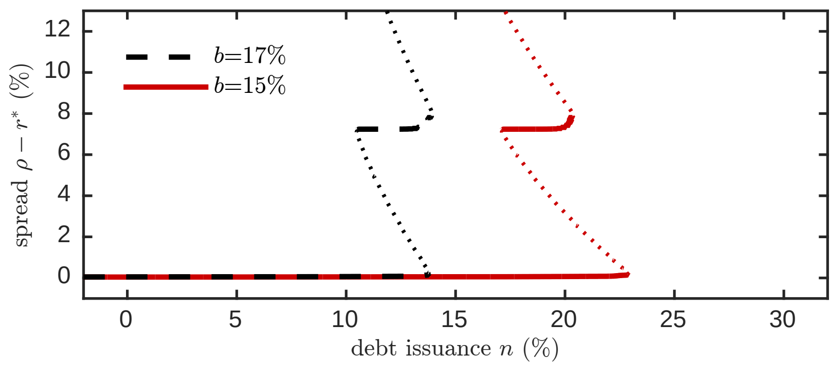
The role of the inherited debt The comparative statics with respect to the initial value of b is similar as in the case of Argentina: the schedule shifts to the left as the debt services b increase. However, we think it's interesting to consider in detail the counterfactual in which, rather than starting with debt services of $b = 15\%$ as in 2012, Spain starts with debt services of $b = 17\%$ as in the ergodic mean of the model (see Table 4 below). Thus, Figure 9b shows the low-growth schedule for a debt service of $b = 17\%$, together with the benchmark of $b = 15\%$ in Figure 8. For the case of $b = 17\%$, the region of multiplicity occurs for levels of debt issuance between $n = 11\%$ and $n = 15\%$ of output, which would involve current account surpluses, since 17% of output must be rolled over. These numbers imply that because the debt obligations were lower than the average of the invariant distribution, Spain managed to handle the crisis substantially

Figure 9: Interest rate spreads for Spain in the low growth state ($b = 15\%$): Comparative statics

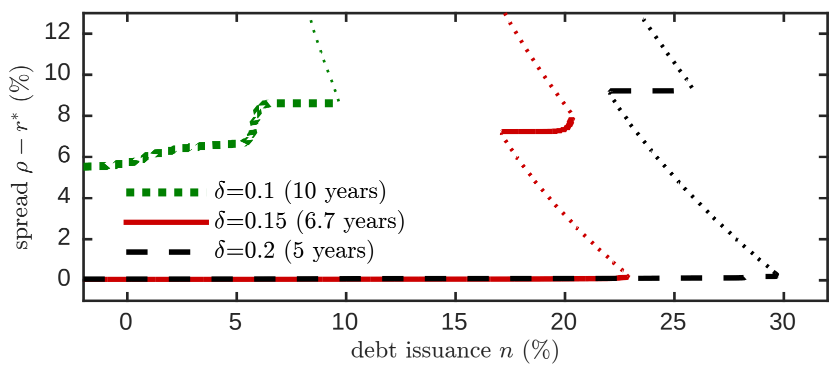
(a) The role of expectation p_B



(b) Inherited debt b



(c) Maturity δ ($b/\delta = 100\%$)



better and run deficits while expectations were good.

If the probability of the bad sunspot was higher, the country would have faced higher spreads even for very low levels of debt issuance. In our narrative, this implies that, with pessimistic expectations, Spain would have only been able to borrow at very high spreads, regardless of the realization of the sunspot.

The role of average maturity Figure 9c reports the schedules for different levels of debt maturity δ. The results intuition is as before: shorter maturity ameliorates the debt-dilution problem and moves the multiplicity region further to the right, while longer maturity strengthens the debt-dilution problem and increases spreads even for low issuance levels.

5.3 The simulated economies

We now show the moments of the simulated economy, for both calibrations, and discuss how they change as we vary the probability of the sunspot.

Argentina Table 3 shows the moments for the simulated economy for Argentina. The first block lists unconditional first moments. The second and third blocks show first moments conditional on the growth state. Finally, the fourth block lists unconditional second moments. The columns represent two different parameterizations of the economy. The first column shows the moments for the benchmark calibration, and the second column shows the effect of reducing the probability of the bad sunspot to $p_B = 1\%$. All moments are computed during periods where the country is not in default.

The main takeaway from Table 3 is that the probability of the sunspot plays a crucial role in driving up average spreads, particularly in the low-growth state, and affecting borrowing choices. As the sunspot probability goes down from 25% to 1%, default rates decrease from 5.1% to 0.5%, while spreads decrease from 16.4% to 0.2%. That is, without any changes in fundamentals, only in beliefs, a borrower mutates from a serial defaulter to a virtually non-defaulter.

Endogenous austerity and gambling for redemption play a key role in producing the simulated moments for Argentina. Both can be observed in the behavior of the policy function for new debt issuance (Figure 10, top panel) and the corresponding spreads (Figure 10, bottom panel). The left column shows results for the low-growth state, while the right panel shows results for the high-growth state.

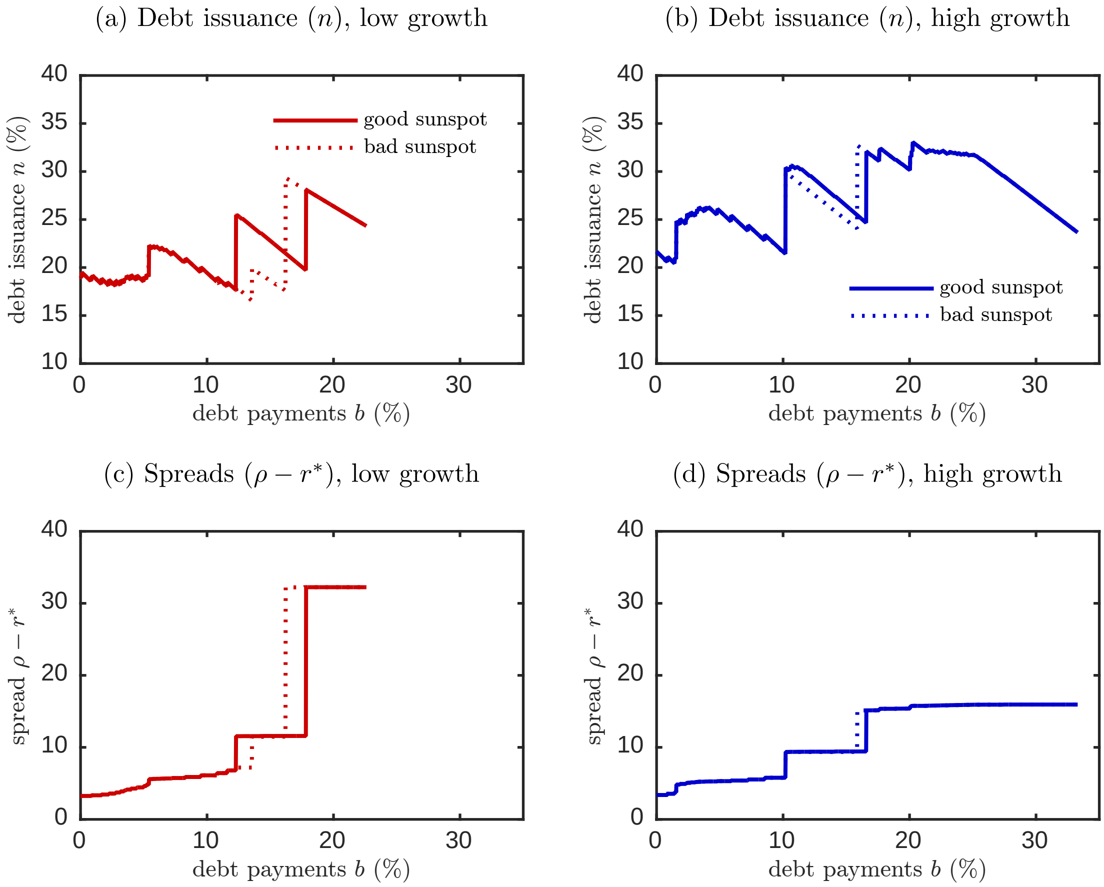
Endogenous austerity is the reason why new debt issuance n declines as the outstanding debt services b increase. Even if outstanding debt services are higher, the borrower reduces new issuance in order to avoid discrete jumps in interest rates, thus cutting down consumption sharply. These interest rate jumps depend on future debt services and,

Table 3: Simulation moments: Argentina

	Benchmark ($p_B = 25\%$)	$p_B = 1\%$
First moments (%)		
avg(b/y)	30	27
avg($qb/\delta y$)	49	63
avg(f/y)	50	61
avg(n/y)	25	25
avg(tb/y)	5.1	1.9
default rate	5.1	0.5
avg($spread$)	16.4	0.2
Low-growth state		
avg(b/y)	22	28
avg($qb/\delta y$)	11	65
avg(f/y)	37	63
avg(n/y)	25	23
avg(tb/y)	-3.0	4.4
default rate	13.3	1.3
avg($spread$)	30.4	0.3
High-growth state		
avg(b/y)	30	26
avg($qb/\delta y$)	50	61
avg(f/y)	50	59
avg(n/y)	25	26
avg(tb/y)	5.4	0.4
default rate	0.0	0.0
avg($spread$)	15.9	0.2
Second moments		
std($spread$) (p.p.)	2.9	0.3
std(c)/std(y) (p.p.)	2.9	1.5

Note: b denotes total debt service, qb/δ denotes the market value of debt, f denotes the face value of debt, n denotes debt issuance, tb denotes trade balance, and y denotes output. We simulate the model economy for 20,000 periods, exclude the first 1,000 periods, and compute the moments conditional on not being in default. The default rate is the number of default episodes per 100 periods divided by 100.

Figure 10: Policy functions and equilibrium spreads for Argentina



therefore, occur for lower levels of n as b increases—see Argentina schedule in Figure 7b. Gambling for redemption, instead, is the mechanism behind the sharp increases in new issuance n. The borrower chooses to bear higher borrowing costs in order to increase consumption and issues further debt in a region where the schedule is largely unresponsive to a further increase in issuance.

The simulations for Argentina in the benchmark calibration ($p_B = 25\%$) visit the region of debt service levels where spreads are high because of expectations. Both endogenous austerity and gambling for redemption are at play there, resulting in high equilibrium spreads and default rates, in particular in periods of low growth.

Expectations play a key role: with more optimistic expectations ($p_B = 1\%$), endogenous austerity would have prevailed for Argentina, inducing low spreads and default probabilities, as the second column of Table 3 shows.

The sunspot also significantly affects second moments, as the last rows in Table 3 show. In particular, as p_B moves from 25% to 1%, the volatility of spreads decreases by a factor of 10, while the volatility of consumption halves.

Spain The moments for the simulated economy for Spain are in Table 4, which is the analog of Table 3 for Argentina, for the optimistic benchmark ($p_B = 1\%$) and for the case of pessimistic expectations ($p_B = 25\%$).

In the low-growth state, the average debt service b is around 16% of GDP. Just rolling over the debt implies borrowing in the multiplicity region, as Figure 9b shows. Yet, as Table 4 also shows, spreads in the low-growth state are essentially zero, while default rates are high. The reason is that more than 99.3% of default episodes occur in high-growth to low-growth transitions. While in the low-growth state, endogenous austerity prevails and there are almost no defaults.

This can be seen in Figure 11, which shows debt issuance policies and the corresponding equilibrium spreads. The debt issuance policy function in the low (high) growth state is depicted in solid red (blue) for the good sunspot and dashed red (blue) for the bad sunspot. The bottom row in Figure 11 shows the corresponding equilibrium spreads. As discussed before, endogenous austerity is the reason why issuance n declines as debt services b increase.

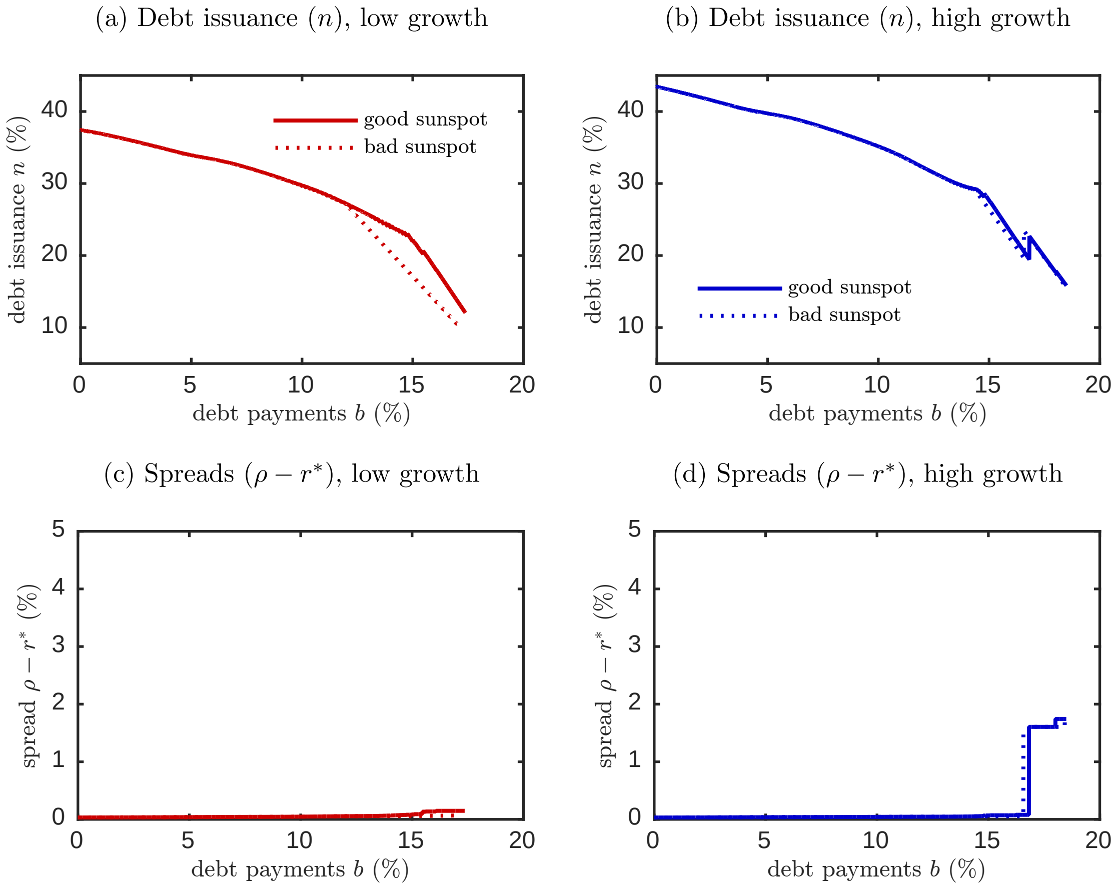
In the low-growth state, the policy function exhibits strong endogenous austerity, with issuance declining as debt services increase. The realization of the bad sunspot induces even stronger endogenous austerity. Overall, default rates are close to zero under low growth, and thus equilibrium spreads are low.

All this implies that while just rolling-over debt services in the low-growth state leads to borrowing in the multiplicity region, that never happens in equilibrium. Endogenous austerity keeps the economy below the multiplicity region so that high spreads due to bad expectations do not happen in equilibrium in the low-growth state. Thus, while there is multiplicity of spreads for the debt service levels Spain faced in 2012 (Figure 8), the simulated economy does not give rise to those high spreads.

The actual experience of Spain, and other European countries, was to implement austerity measures in response to the observed high spreads. In the model, the high spreads triggering endogenous austerity happen off equilibrium, so they are not observed in the simulations. Thus, our model is only partially successful in explaining the events in those countries during the sovereign debt crisis period.

We think that with reasonable/small changes to our model, Calvo-type multiplicity could still explain the jump in Spain's spreads in 2012. Note that, as Table 4 also shows, with pessimistic expectations ($p_B = 25\%$), Spain would have faced much higher spreads in equilibrium, even for low levels of issuance (Figure 9a). We conjecture that stochastic changes in p_B, as in Bocola and Dovis (2019), may induce a rise in spreads as the result of expectations becoming more pessimistic over time. That is, a gradual increase in p_B could lead to spreads increasing to almost 7%, as observed for Spain during July 2012. Additionally, beyond utility losses, our model has no costs associated with sudden declines in consumption, which may be unrealistic for public/government

Figure 11: Policy functions and equilibrium spreads for Spain



spending that is typically harder to quickly adjust. Adding such sluggish consumption responses could lead to a lengthening of the fiscal austerity measures and, thus, to high spreads in equilibrium.

Table 4: Simulation moments conditional on not being in default: Spain

	Benchmark ($p_B = 1\%$)	$p_B = 25\%$
First moments (%)		
avg(b/y)	17	14
avg($qb/\delta y$)	88	53
avg(f/y)	86	52
avg(n/y)	18	12
avg(tb/y)	-1.4	1.6
default rate	5.2	6.1
avg($spread$)	1.2	7.7
Low-growth state		
avg(b/y)	16	12
avg($qb/\delta y$)	80	35
avg(f/y)	84	47
avg(n/y)	17	12
avg(tb/y)	-0.9	0.5
default rate	18	20
avg($spread$)	0.1	9.7
High-growth state		
avg(b/y)	17	14
avg($qb/\delta y$)	89	55
avg(f/y)	86	53
avg(n/y)	18	12
avg(tb/y)	-1.5	1.8
default rate	0	0
avg($spread$)	1.4	7.5
Second moments		
std($spreads$) (p.p)	0.7	1.5
std(c)/std(y) (p.p)	2.7	2.3

Note: b denotes total debt service (principal+coupon payments), qb/δ denotes the market value of debt, f denotes the face value of debt, n denotes debt issuance, tb denotes trade balance, and y denotes output. We simulate the model economy for 20,000 periods, exclude the first 1,000 periods, and compute the moments conditional on not being in default. The default rate is the number of default episodes per 100 periods divided by 100.

6 Conclusion

In the model of sovereign debt crises of Calvo (1988), there are multiple interest rate schedules because expectations of high probabilities of default are self-confirming. In particular, if expectations of default are high, interest rates must be high, and high interest rates increase the burden of debt, inducing the borrower to default. The question that remains is whether the source of multiplicity is quantitatively relevant. In particular, we are interested in determining the role that it may have played in the sovereign debt crises of Argentina in 2001 and Southern Europe in the early 2010s.

We argue that the mechanism in Calvo (1988) is quantitatively relevant and that key for multiplicity is a bimodal output process with persistent good and bad times. We estimate this output process for a set of countries that have recently been exposed to sovereign debt crises. We show that a sunspot realization can induce discrete jumps in interest rates even with no change in fundamentals. These expectations-driven jumps in interest rates can only occur during stagnations. Interest rate jumps, either because of expectations or fundamentals, can induce either endogenous austerity, in which the borrower refrains from borrowing to avoid the jump in rates, or gambling for redemption, in which the borrower increases debt beyond the jump in rates.

We consider two calibrations of the model, one targeted to Argentina and another to Spain. We show that the Argentine calibration generates equilibrium paths that resemble the events of the 2001 debt crisis, with credit spreads going up by magnitudes similar to the observed ones. In the calibration for Spain, expectations-driven high rates induce austerity measures such as those observed in the early 2010s. As a result of this endogenous austerity, the high spreads remain off equilibrium for Spain. Endogenous austerity and gambling for redemption are central in explaining our findings.

A key takeaway of our paper is that expectations, as measured by the sunspot probability of selecting an interest rate schedule, have a large quantitative effect on model outcomes. Assuming optimistic expectations for Argentina results in substantially lower default rates and credit spreads, even with no change in fundamentals. Similarly, assuming pessimistic expectations for Spain results in substantially higher default rates and credit spreads. Thus, expectations are a major driver explaining default rates and credit spread differences between Spain and Argentina.

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A Endowment process estimation

In this appendix, we provide more details on the estimation of the GDP process used in the quantitative evaluation of the model. We use the filter in Kim (1994) to obtain the likelihood function and explore the posterior using a Metropolis-Hastings MCMC algorithm with a random walk proposal density.

Let Y_t denote the country's GDP during year t, and let $\Delta y_t = \log(Y_t) - \log(Y_{t-1})$ denote GDP growth. The process in equations (7)–(8) implies

$$\Delta y_t = g_t + \sigma\left(\epsilon_t - \epsilon_{t-1}\right) \tag{A.1}$$

where $\epsilon_t \sim \mathcal{N}(0,1)$, and g_t follows a two-state Markov process with transition probabilities $p_g(g'|g)$. Denote g_L and g_H to the possible values of g: $g_t \in \{g_L, g_H\}$. The transition probability is fully summarized by the two parameters p_L and p_H, where $p_L = p_g\left(g_{t+1} = g_L | g_t = g_L\right)$ and $p_H = p_g\left(g_{t+1} = g_H | g_{it} = g_H\right)$.

Let $\theta = \{g_L, g_H, \sigma, p_L, p_H\}$ collect all parameters determining the process in equation (A.1). Denote $\mathcal{Y}_t = \{\Delta y_0, \Delta y_1, \ldots, \Delta y_t\}$ to be all observations up to period t, and $\mathcal{L}(\theta|\mathcal{Y}_T)$ to be the likelihood of parameters θ where T is the total number of observations. We construct the likelihood $\mathcal{L}(\theta|\mathcal{Y}_T)$ using the filter in Kim (1994).

We assume uniform priors on parameters, which bounds the possible space for θ (see Table 1). Additionally, we include the normalization of $g_L \leq g_H$ as part of our priors. Let $p(\theta)$ denote the prior selection. The posterior of parameters θ is then given by

$$\mathcal{P}(\theta|\mathcal{Y}_T) = \mathcal{L}(\theta|Y_T)p(\theta) \quad . \tag{A.2}$$

We explore the posterior $\mathcal{P}(\theta|\mathcal{Y}_T)$ using a Metropolis-Hastings MCMC algorithm with a random walk as a proposal density. In particular, the proposal density is a normal $\mathcal{N}\left(\theta_{n-1}, \bar{\sigma}\Sigma_\theta\right)$, where θ_{n-1} is the last draw of the chain, Σ_θ has $\theta^* = \arg\max_\theta \mathcal{P}\left(\theta|\mathcal{Y}_T\right)$ in its diagonal and zeros otherwise, and $\bar{\sigma}$ is selected so that the rejection rate in the chain is between 60% and 70%. We simulate 10 chains of length 125,000 each and compute posteriors by pooling 1 out of every 10 draws from the last 100,000 observations in each chain. Table 1 contains all posterior estimates.

Data are from the Conference Board Total Economy Database, and we used GDP per capita in 2016 U.S. dollars (converted to 2016 price level with updated 2011 PPPs) as our measure of output.

Different standard deviations across states We also estimate the endowment process for the case in which the standard deviation of the temporary shock depends on whether the economy is at the low-growth state, $g_t = g_L$, or the high-growth state, $g_t = g_H$. We denote the standard deviations in the low- and high-growth states by σ_L

Table A.1: Prior and posterior distributions: different σ's across growth states

	$\ln(g_L)$	$\ln(g_H)$	p_L	p_H	σ_L	σ_H
	Prior distribution					
	$U[-0.1, 0.1]$	$U[-0.1, 0.1]$	$U[0.1, 1.0]$	$U[0.1, 1.0]$	$U[10^{-3}, 0.5]$	$U[10^{-3}, 0.5]$
Countries	**Posterior distribution** (mean, and 5th to 95th percentile intervals)					
Italy	-0.014	0.026	0.654	0.766	0.020	0.011
	[-0.022,-0.008]	[0.019,0.030]	[0.356,0.974]	[0.523,0.990]	[0.013,0.031]	[0.006,0.020]
Portugal	-0.002	0.048	0.790	0.703	0.020	0.018
	[-0.011,0.003]	[0.040,0.058]	[0.503,0.976]	[0.403,0.990]	[0.012,0.029]	[0.011,0.029]
Spain	-0.017	0.035	0.608	0.810	0.019	0.015
	[-0.025,-0.009]	[0.027,0.040]	[0.325,0.866]	[0.618,0.978]	[0.009,0.031]	[0.009,0.025]
Argentina	-0.027	0.072	0.776	0.570	0.046	0.013
	[-0.043,-0.023]	[0.057,0.076]	[0.562,0.916]	[0.344,0.783]	[0.034,0.075]	[0.008,0.024]
Brazil	-0.032	0.029	0.605	0.790	0.022	0.019
	[-0.071,-0.019]	[0.026,0.032]	[0.059,0.874]	[0.619,0.926]	[0.011,0.063]	[0.014,0.026]

Note: For each country, we estimate an output process as: $\Delta \ln y_t = \ln(g_t) + \sigma_t \epsilon_t - \sigma_{t-1}\epsilon_{t-1}$, in which $\epsilon_t \sim N(0, 1)$ and $g_t \in \{g_L, g_H\}$, with $\Pr(g_{t+1} = g_L | g_t = g_L) = p_L$ and $\Pr(g_{t+1} = g_H | g_t = g_H) = p_H$. If $g_t = g_L$, then $\sigma_t = \sigma_L$, otherwise $\sigma_t = \sigma_H$. The table reports the mean and the interval between the 5th and 95th percentiles of the posterior distributions of each of the parameters for each country. The table also reports the prior distributions we used, which were chosen to be the same across countries. For each country, we use data on GDP per capita in 2016 US$ (converted to 2016 price level with updated 2011 PPPs) between 1980 and 2017 from the Conference Board Total Economy Database as the measure of y_t.

and σ_H, respectively. Table A.1 shows the estimates for all countries. First, note that the standard deviation posterior mean is similar across growth states, except for the case of Argentina, and that the mean of σ_L is larger than the mean of σ_H in all cases. In the case of Italy, the difference between σ's is somewhat larger, but the values are still within the interval analyzed in Figures E.1c and E.2c, which are shown to not have significant implications for our results. Finally, Table A.1 also shows that the posterior distributions of the remaining parameters are very similar to their counterparts in Table 1.

B Data appendix

In this appendix, we describe the data sources we used to compute the series of debt levels and debt services for Argentina and Spain.

Argentina:

Debt level We use the data on gross external debt reported by the World Bank's International Debt Statistics (series code DT.DOD.DECT.GN.ZS). This measure corresponds to the principal of bonds issuance, at face value, summed over all outstanding bonds.

Debt service Computed as the sum of external short-term debt in the previous year and current debt service on external long-term debt. Data on short-term debt (series code DT.DOD.DSTC.ZS) and on debt service on external long-term debt (series code DT.TDS.DECT.CD) are from the World Bank's International Debt Statistics.

Data from the World Bank's International Debt Statistics are expressed as a percentage of GNI. We convert them to percentage of GDP using the series of the ratio of GNP to GDP for Argentina from the University of Pennsylvania, retrieved from FRED, Federal Reserve Bank of St. Louis (series code GNPGDPARA156NUPN).

Spain:

Debt level We use the Net International Investment Position reported by Banco de España (series code BE_17_ 30.7).

Debt service Data on debt service for Spain are not readily available. We impute debt services as the sum of: interest payments due in the year, short-term debt maturing in the year, and the fraction of long-term debt that matures in the year.

- *Interest payments*: We use data on net investment income from the balance of payments. It is computed as investment payments (series code BE_17_5A.7) minus investment income (series code BE_17_5A.6), both from Banco de España.
- *Short-term debt*: We use data on net short-term debt related to portfolio income excluding Banco de España. It is computed as the liabilities (series code BE_17_27.8) minus assets (series code BE_17_22.8), both from Banco de España.
- *Long-term debt maturing*: We compute the stock of long-term debt as the debt level minus the short-term debt, both as described above. We multiply this amount by $\delta = 0.15$ to estimate the principal payments on long-term debt.

We do not have data on average maturity for the net international investment position of Spain. In turn, we use $\delta = 0.15$ based on the average maturity of Spain's central government debt of 6 to 7 years, as reported by the Tesoro Público de España.

C Model normalization

In this appendix, we provide more details on the normalization step. First, we show that the value functions and the interest rate schedule satisfy certain homogeneity properties. Second, we use these homogeneity properties to derive the stationary system presented in Section 3.2. The algorithm for solving this stationary system is discussed in Appendix D.

We proceed by guess and verify. In particular, we guess that equilibrium functions satisfy certain homogeneity properties and then use the equilibrium equations to verify that our guess is consistent with the equilibrium definition. That is, we do not claim that all possible equilibria satisfy these properties, but rather that the equilibrium admits a solution with these properties.

Proposition 1 (Homogeneity of equilibrium functions)**.** *For any $\lambda > 0$, the equilibrium functions admit a solution that satisfies the following homogeneity properties: (i) $V^{nd}(\lambda B, \lambda\Gamma_-, g, s, \epsilon) = \lambda^{1-\gamma}V^{nd}(B, \Gamma_-, g, s, \epsilon)$ and $V^d(\lambda B, \lambda\Gamma_-, g, s, \epsilon) = \lambda^{1-\gamma}V^d(B, \Gamma_-, g, s, \epsilon)$; (ii) $\mathbf{B}'(\lambda B, \lambda\Gamma_-, g, s, \epsilon) = \lambda\mathbf{B}'(B, \Gamma_-, g, s, \epsilon)$; (iii) $\mathcal{Q}(B, \lambda R, \Gamma_-, g, s) = \lambda\mathcal{Q}(B, R, \Gamma_-, g, s)$ and $\mathcal{X}(B, \lambda R, \Gamma_-, g, s) = \lambda\mathcal{X}(B, R, \Gamma_-, g, s)$; (iv) $\mathcal{Q}(\lambda B, R, \lambda\Gamma_-, g, s) = \mathcal{Q}(B, R, \Gamma_-, g, s)$ and $\mathcal{X}(\lambda B, R, \lambda\Gamma_-, g, s) = \mathcal{X}(B, R, \Gamma_-, g, s)$; and (v) $\mathcal{R}(\lambda N, \lambda B, \lambda\Gamma_-, g, s) = \mathcal{R}(N, B, \Gamma_-, g, s)$.*

Proof. Let's start with (i). From equation (16), the value of default $V^d(\cdot)$ can be written as

$$\begin{aligned} V^d(\lambda B, \lambda\Gamma_-, g, s, \epsilon) &= \frac{[\phi(g)g\lambda\Gamma_- e^{\sigma\epsilon}]^{1-\gamma}}{1-\gamma} + \beta\mathbb{E}\Bigg[\theta V^d(\lambda B, g\lambda\Gamma_-, g', s', \epsilon') \\ &+ (1-\theta)\max\begin{Bmatrix} V^{nd}(\kappa\lambda B, g\lambda\Gamma_-, g', s', \epsilon'), \\ V^d(\lambda B, g\lambda\Gamma_-, g', s', \epsilon') \end{Bmatrix} |\Gamma_-, g, s\Bigg] \\ &= \lambda^{1-\gamma}\frac{[\phi(g)g\Gamma_- e^{\sigma\epsilon}]^{1-\gamma}}{1-\gamma} + \beta\mathbb{E}\Bigg[\theta\lambda^{1-\gamma}V^d(B, g\Gamma_-, g', s', \epsilon') \\ &+ (1-\theta)\max\begin{Bmatrix} \lambda^{1-\gamma}V^{nd}(\kappa B, g\Gamma_-, g', s', \epsilon'), \\ \lambda^{1-\gamma}V^d(B, g\Gamma_-, g', s', \epsilon') \end{Bmatrix} |\Gamma_-, g, s\Bigg] \\ &= \lambda^{1-\gamma}V^d(B, \Gamma_-, g, s, \epsilon) \end{aligned}$$

where the second equality uses the guess for $V^{nd}(\cdot)$ and $V^d(\cdot)$, and the last equality verifies the guess for $V^d(\cdot)$.

Next, we verify the homogeneity property for the value of no default $V^{nd}(\cdot)$. For this purpose, it's convenient to express the issuance limit $\bar{N}(B, \Gamma_-, g, s)$ in (15) as set constraint. In particular, let $\mathbb{N}(B, \Gamma_-, g, s)$ be the set of issuance N such that next period default probability is below $\bar{p}$. Formally

$$\mathbb{N}(B, \Gamma_-, g, s) = \begin{Bmatrix} N: & \mathbb{P}\left(V^{nd}(B', \Gamma, g', s', \epsilon') \leq V^d(B', \Gamma, g', s', \epsilon') \middle| \Gamma_-, g, s\right) \leq \bar{p} \\ \text{s. to} & B' = (1-\delta)B + \mathcal{R}(N, B, \Gamma_-, g, s)N \end{Bmatrix} \tag{C.1}$$

Thus, the set $\mathbb{N}(B, \Gamma_-, g, s)$ contains all possible issuance N such that the default probability next periods is below a number $\bar{p}$. Under our guess, the set $\mathbb{N}(\cdot)$ satisfies that if $N \in \mathbb{N}(B, \Gamma_-, g, s)$, then $\lambda N \in \mathbb{N}(\lambda B, \lambda\Gamma_-, g, s)$. To see this, note that, when the state is $(\lambda B, \lambda\Gamma_-, g, s)$, the probability of default after issuance λN is given as

$$\mathbb{P}\left(V^{nd}\begin{pmatrix}(1-\delta)\lambda B + \mathcal{R}(\lambda N, \lambda B, \lambda\Gamma_-, g, s)\lambda N, \\ \lambda\Gamma, g', s', \epsilon'\end{pmatrix} \leq V^{d}\begin{pmatrix}(1-\delta)\lambda B + \mathcal{R}(\lambda N, \lambda B, \lambda\Gamma_-, g, s)\lambda N, \\ \lambda\Gamma, g', s', \epsilon'\end{pmatrix} \middle| \Gamma_-, g, s\right)$$
$$= \mathbb{P}\left(V^{nd}\begin{pmatrix}(1-\delta)\lambda B + \mathcal{R}(N, B, \Gamma_-, g, s)\lambda N, \\ \lambda\Gamma, g', s', \epsilon'\end{pmatrix} \leq V^{d}\begin{pmatrix}(1-\delta)\lambda B + \mathcal{R}(N, B, \Gamma_-, g, s)\lambda N, \\ \lambda\Gamma, g', s', \epsilon'\end{pmatrix} \middle| \Gamma_-, g, s\right)$$
$$= \mathbb{P}\left(\lambda^{1-\gamma} V^{nd}\begin{pmatrix}(1-\delta) B + \mathcal{R}(N, B, \Gamma_-, g, s) N, \\ \Gamma, g', s', \epsilon'\end{pmatrix} \leq \lambda^{1-\gamma} V^{d}\begin{pmatrix}(1-\delta) B + \mathcal{R}(N, B, \Gamma_-, g, s) N, \\ \Gamma, g', s', \epsilon'\end{pmatrix} \middle| \Gamma_-, g, s\right)$$
$$= \mathbb{P}\left(V^{nd}\begin{pmatrix}(1-\delta) B + \mathcal{R}(N, B, \Gamma_-, g, s) N, \\ \Gamma, g', s', \epsilon'\end{pmatrix} \leq V^{d}\begin{pmatrix}(1-\delta) B + \mathcal{R}(N, B, \Gamma_-, g, s) N, \\ \Gamma, g', s', \epsilon'\end{pmatrix} \middle| \Gamma_-, g, s\right)$$
$$= \mathbb{P}\left(V^{nd}(B', \Gamma, g', s', \epsilon') \leq V^{d}(B', \Gamma, g', s', \epsilon') \middle| \Gamma_-, g, s\right)$$

where the second line uses our guess for $\mathcal{R}(\cdot)$, and the third line uses the guess for $V^{nd}(\cdot)$ and $V^d(\cdot)$. The last line shows that the probability of default after issuance λN under state $(\lambda B, \lambda\Gamma_-, g, s)$ is the same as the probability of default after issuance N under state (B, Γ_-, g, s). Thus, if $N \in \mathbb{N}(B, \Gamma_-, g, s)$, then $\lambda N \in \mathbb{N}(\lambda B, \lambda\Gamma_-, g, s)$.

Let $\mathcal{U}(N, B, \Gamma_-, g, s, \epsilon)$ be the value of no default when issuing N. That is

$$\mathcal{U}(N, B, \Gamma_-, g, s, \epsilon) = \left\{ \frac{C^{1-\gamma}}{1-\gamma} + \beta\mathbb{E}\left[\max\left\{V^{nd}(B', \Gamma, g', s', \epsilon'), V^{d}(B', \Gamma, g', s', \epsilon')\right\} |\Gamma_-, g, s\right] \right\}$$
$$s.t. \tag{C.2}$$
$$C + B = Y + N,$$
$$B' = (1-\delta)B + \mathcal{R}(N, B, \Gamma_-, g, s)N,$$
$$Y = \Gamma e^{\sigma\epsilon}, \quad \Gamma = e^g\Gamma_-$$

When defining $\mathcal{U}(\cdot)$, we imposed all constrains of $V^{nd}(\cdot)$ in equation (15) except for issuance limit $N \in \mathbb{N}(B, \Gamma_-, g, s)$. Thus, we have

$$V^{nd}(B, \Gamma_-, g, s, \epsilon) = \max_N \left\{\mathcal{U}(N, B, \Gamma_-, g, s, \epsilon) \;\text{ s. to }\; N \in \mathbb{N}(B, \Gamma_-, g, s)\right\}. \tag{C.3}$$

Notice that $\mathcal{U}(\lambda N, \lambda B, \lambda\Gamma_-, g, s, \epsilon) = \lambda^{1-\gamma}\mathcal{U}(N, B, \Gamma_-, g, s, \epsilon)$. In particular,

$$\mathcal{U}(\lambda N, \lambda B, \lambda\Gamma_-, g, s, \epsilon) = \frac{[g\lambda\Gamma_- e^{\sigma\epsilon} + \lambda N - \lambda B]^{1-\gamma}}{1-\gamma}$$
$$+ \beta\mathbb{E}\left[\max\left\{\begin{matrix} V^{nd}\Big((1-\delta)\lambda B + \mathcal{R}(\lambda N, \lambda B, \lambda\Gamma_-, g, s)\lambda N, \lambda g\Gamma_-, g', s', \epsilon'\Big) \\ V^{d}\Big((1-\delta)\lambda B + \mathcal{R}(\lambda N, \lambda B, \lambda\Gamma_-, g, s)\lambda N, \lambda g\Gamma_-, g', s', \epsilon'\Big)\end{matrix}\right\} |\Gamma_-, g, s\right]$$
$$= \lambda^{1-\gamma}\frac{[g\Gamma_- e^{\sigma\epsilon} + N - B]^{1-\gamma}}{1-\gamma}$$
$$+ \beta\mathbb{E}\left[\max\left\{\begin{matrix} \lambda^{1-\gamma}V^{nd}\Big((1-\delta)B + \mathcal{R}(N, B, \Gamma_-, g, s)N, g\Gamma_-, g', s', \epsilon'\Big) \\ \lambda^{1-\gamma}V^{d}\Big((1-\delta)B + \mathcal{R}(N, B, \Gamma_-, g, s)N, g\Gamma_-, g', s', \epsilon'\Big)\end{matrix}\right\} |\Gamma_-, g, s\right]$$
$$= \lambda^{1-\gamma}\mathcal{U}(N, B, \Gamma_-, g, s, \epsilon) \tag{C.4}$$

where the second equality uses the guess on $V^d(\cdot)$, $V^{nd}(\cdot)$, and $\mathcal{R}(\cdot)$.

We use equation (C.3) to show that the issuance policy is linear in (B, Γ_-). Let $\mathbf{N}(B, \Gamma_-, g, s, \epsilon)$ be the optimal policy in equation (C.3). Then, under our guess, we have $\mathbf{N}(\lambda B, \lambda\Gamma_-, g, s, \epsilon) = \lambda\mathbf{N}(B, \Gamma_-, g, s, \epsilon)$. We proceed to prove this by contradiction. Assume that there is another policy, $\lambda\hat{N} \neq \lambda\mathbf{N}(B, \Gamma_-, g, s, \epsilon)$, which is feasible and delivers higher utility $\mathcal{U}$. That is, $\lambda\hat{N} \in \mathbb{N}(\lambda B, \lambda\Gamma_-, g, s)$ and $\mathcal{U}\left(\lambda\hat{N}, \lambda B, \lambda\Gamma_-, g, s, \epsilon\right) > \mathcal{U}\left(\lambda\mathbf{N}(B, \Gamma_-, g, s, \epsilon), \lambda B, \lambda\Gamma_-, g, s, \epsilon\right)$. Using the homogeneity properties of the set $\mathbb{N}(\cdot)$, we know that if $\lambda\hat{N} \in \mathbb{N}(\lambda B, \lambda\Gamma_-, g, s)$, then $\hat{N} \in \mathbb{N}(B, \Gamma_-, g, s)$. Using the homogeneity of $\mathcal{U}$ in equation (C.4), we know that if $\mathcal{U}\left(\lambda\hat{N}, \lambda B, \lambda\Gamma_-, g, s, \epsilon\right) > \mathcal{U}\left(\lambda\mathbf{N}(B, \Gamma_-, g, s, \epsilon)\lambda B, \lambda\Gamma_-, g, s, \epsilon\right)$, then $\mathcal{U}\left(\hat{N}, B, \lambda\Gamma_-, g, s, \epsilon\right) > \mathcal{U}\left(\mathbf{N}(B, \Gamma_-, g, s, \epsilon), B, \Gamma_-, g, s, \epsilon\right)$. This is a contradiction, because $\hat{N}$ would be feasible under state $(B, \Gamma_-, g, s, \epsilon)$ and deliver higher utility than $\mathbf{N}(B, \Gamma_-, g, s, \epsilon)$, contradicting that $\mathbf{N}(B, \Gamma_-, g, s, \epsilon)$ is an optimal policy. Then, we have that $\mathbf{N}(\lambda B, \lambda G_-, g, s, \epsilon) = \lambda\mathbf{N}(B, G_-, g, s, \epsilon)$.

Finally, we can show the homogeneity property of $V^{nd}(\cdot)$. In particular,

$$
\begin{aligned}
V^{nd}(\lambda B, \lambda G_-, g, s, \epsilon) &= \mathcal{U}(\mathbf{N}(\lambda B, \lambda G_-, g, s, \epsilon), \lambda B, \lambda G_-, g, s, \epsilon) \\
&= \mathcal{U}(\lambda\mathbf{N}(B, G_-, g, s, \epsilon), \lambda B, \lambda G_-, g, s, \epsilon) \\
&= \lambda^{1-\gamma}\mathcal{U}(\mathbf{N}(B, G_-, g, s, \epsilon), B, G_-, g, s, \epsilon) \\
&= \lambda^{1-\gamma}V^{nd}(B, \Gamma_-, g, s, \epsilon) \qquad \text{(C.5)}
\end{aligned}
$$

where the second line uses the linearity of $\mathbf{N}(\cdot)$, the third line uses the homogeneity of $\mathcal{U}(\cdot)$, and the first and last lines use the definition of $\mathcal{U}(\cdot)$.

Let's move to (ii). We have that

$$
\mathbf{B}'(B, \Gamma_-, g, s, \epsilon) = (1-\delta)B + \mathcal{R}(\mathbf{N}(B, \Gamma_-, g, s, \epsilon), B, \Gamma_-, g, s, \epsilon)\mathbf{N}(B, \Gamma_-, g, s, \epsilon).
$$

Then,

$$
\begin{aligned}
\mathbf{B}'(\lambda B, \lambda\Gamma_-, g, s, \epsilon) &= (1-\delta)\lambda B + \mathcal{R}(\mathbf{N}(\lambda B, \lambda\Gamma_-, g, s, \epsilon), \lambda B, \lambda\Gamma_-, g, s, \epsilon)\mathbf{N}(\lambda B, \lambda\Gamma_-, g, s, \epsilon) \\
&= (1-\delta)\lambda B + \mathcal{R}(\lambda\mathbf{N}(B, \Gamma_-, g, s, \epsilon), \lambda B, \lambda\Gamma_-, g, s, \epsilon)\lambda\mathbf{N}(B, \Gamma_-, g, s, \epsilon) \\
&= (1-\delta)\lambda B + \mathcal{R}(\mathbf{N}(B, \Gamma_-, g, s, \epsilon), B, \Gamma_-, g, s, \epsilon)\lambda\mathbf{N}(B, \Gamma_-, g, s, \epsilon) \\
&= \lambda\mathbf{B}'(B, \Gamma_-, g, s, \epsilon) \qquad \text{(C.6)}
\end{aligned}
$$

where the second line uses the linearity of $\mathbf{N}(\cdot)$, the third line uses our guess for $\mathcal{R}(\cdot)$, and the fourth line confirms the linearity of $\mathbf{B}'(\cdot)$.

Let's move to (iii). When proving the homogeneity properties of the price functions $\mathcal{Q}(\cdot)$ and $\mathcal{X}(\cdot)$, we are essentially comparing two bonds—one that pays R, another that pays λR—while keeping the debt services B fixed. Thus, the state of the borrower doesn't change as we move λ.

From equation (19), we have

$$\begin{aligned}
\mathcal{Q}(B,\lambda R,\Gamma_-,g,s,\epsilon) &= [1-\mathbf{D}(B,\Gamma_-,g,s,\epsilon)]\left[\lambda R + \frac{1-\delta}{1+r^*}\mathbb{E}\Big[\mathcal{Q}\left(\mathbf{B}'(B,\Gamma_-,g,s,\epsilon),\lambda R,\Gamma,g',s',\epsilon'\right)|\Gamma_-,g,s\Big]\right] \\
&+ \mathbf{D}(B,\Gamma_-,g,s,\epsilon)\mathcal{X}(B,\lambda R,\Gamma_-,g,s,\epsilon) \\
&= [1-\mathbf{D}(B,\Gamma_-,g,s,\epsilon)]\left[\lambda R + \frac{1-\delta}{1+r^*}\mathbb{E}\Big[\lambda\mathcal{Q}\left(\mathbf{B}'(B,\Gamma_-,g,s,\epsilon),R,\Gamma,g',s',\epsilon'\right)|\Gamma_-,g,s\Big]\right] \\
&+ \mathbf{D}(B,\Gamma_-,g,s,\epsilon)\lambda\mathcal{X}(B,\lambda R,\Gamma_-,g,s,\epsilon) \\
&= \lambda\mathcal{Q}(B,R,\Gamma_-,g,s,\epsilon) \qquad \text{(C.7)}
\end{aligned}$$

where the second line uses our guess for $\mathcal{Q}(\cdot)$ and $\mathcal{X}(\cdot)$, and the third line verifies the guess for $\mathcal{Q}(\cdot)$. Mutatis mutandis, same procedure for $\mathcal{X}(\cdot)$ using equation (20)

$$\begin{aligned}
\mathcal{X}(B,\lambda R,\Gamma_-,g,s,\epsilon) &= \frac{1}{1+r^*}\mathbb{E}\Bigg[\theta\mathcal{X}(B',\lambda R,\Gamma,g',s',\epsilon') + (1-\theta)\left[1-\mathbf{E}(B',\Gamma,g',s',\epsilon')\right]\mathcal{X}(B',\lambda R,\Gamma,g',s',\epsilon') \\
&+ (1-\theta)\mathbf{E}(B',\Gamma,g',s',\epsilon')\mathcal{Q}(\kappa B',\kappa\lambda R,\Gamma,g',s',\epsilon)|\Gamma_-,g,s\Bigg] \\
&= \frac{1}{1+r^*}\mathbb{E}\Bigg[\theta\lambda\mathcal{X}(B',R,\Gamma,g',s',\epsilon') + (1-\theta)\left[1-\mathbf{E}(B',\Gamma,g',s',\epsilon')\right]\lambda\mathcal{X}(B',R,\Gamma,g',s',\epsilon') \\
&+ (1-\theta)\mathbf{E}(B',\Gamma,g',s',\epsilon')\lambda\mathcal{Q}(\kappa B',\kappa R,\Gamma,g',s',\epsilon)|\Gamma_-,g,s\Bigg] \\
&= \lambda\mathcal{X}(B,R,\Gamma_-,g,s,\epsilon) \qquad \text{(C.8)}
\end{aligned}$$

Let's move to (iv). We start by showing that default and re-entry policies satisfy $\mathbf{D}(\lambda B,\lambda\Gamma_-,g,s,\epsilon) = \mathbf{D}(B,\Gamma_-,g,s,\epsilon)$ and $\mathbf{E}(\lambda B,\lambda\Gamma_-,g,s,\epsilon) = \mathbf{E}(B,\Gamma_-,g,s,\epsilon)$. In particular, from equation (17) we have

$$\begin{aligned}
\mathbf{D}(\lambda B,\lambda\Gamma_-,g,s,\epsilon) &= \mathbb{I}\left\{V^{nd}(\lambda B,\lambda\Gamma_-,g,s,\epsilon) < V^d(\lambda B,\lambda\Gamma_-,g,s,\epsilon)\right\} \\
&= \mathbb{I}\left\{\lambda^{1-\gamma}V^{nd}(B,\Gamma_-,g,s,\epsilon) < \lambda^{1-\gamma}V^d(B,\Gamma_-,g,s,\epsilon)\right\} \\
&= \mathbf{D}(B,\Gamma_-,g,s,\epsilon) \qquad \text{(C.9)}
\end{aligned}$$

where $\mathbb{I}\{\cdot\}$ is an indicator function. Similarly, using equation (18)

$$\begin{aligned}
\mathbf{E}(\lambda B,\lambda\Gamma_-,g,s,\epsilon) &= \mathbb{I}\left\{V^{nd}(\kappa\lambda B,\lambda\Gamma_-,g,s,\epsilon) \geq V^d(\lambda B,\lambda\Gamma_-,g,s,\epsilon)\right\} \\
&= \mathbb{I}\left\{\lambda^{1-\gamma}V^{nd}(\kappa B,\Gamma_-,g,s,\epsilon) \geq \lambda^{1-\gamma}V^d(B,\Gamma_-,g,s,\epsilon)\right\} \\
&= \mathbf{E}(B,\Gamma_-,g,s,\epsilon) \qquad \text{(C.10)}
\end{aligned}$$

Then, using equation (19), we have

$$
\begin{aligned}
\mathcal{Q}(\lambda B, R, \lambda\Gamma_-, g, s, \epsilon) &= [1-\mathbf{D}(\lambda B, \lambda\Gamma_-, g, s, \epsilon)]\left[1+\frac{1-\delta}{1+r^*}\mathbb{E}\Big[\mathcal{Q}\left(\mathbf{B}'(\lambda B, \lambda\Gamma_-, g, s, \epsilon), R, \lambda\Gamma, g', s', \epsilon'\right)|\Gamma_-, g, s\Big]\right] \\
&+ \mathbf{D}(\lambda B, \lambda\Gamma_-, g, s, \epsilon)\mathcal{X}(\lambda B, R, \lambda\Gamma_-, g, s, \epsilon) \\
&= [1-\mathbf{D}(\lambda B, \lambda\Gamma_-, g, s, \epsilon)]\left[1+\frac{1-\delta}{1+r^*}\mathbb{E}\Big[\mathcal{Q}\left(\lambda\mathbf{B}'(B, \Gamma_-, g, s, \epsilon), R, \lambda\Gamma, g', s', \epsilon'\right)|\Gamma_-, g, s\Big]\right] \\
&+ \mathbf{D}(\lambda B, \lambda\Gamma_-, g, s, \epsilon)\mathcal{X}(\lambda B, R, \lambda\Gamma_-, g, s, \epsilon) \\
&= [1-\mathbf{D}(\lambda B, \lambda\Gamma_-, g, s, \epsilon)]\left[1+\frac{1-\delta}{1+r^*}\mathbb{E}\Big[\mathcal{Q}\left(\mathbf{B}'(B, \Gamma_-, g, s, \epsilon), R, \Gamma, g', s', \epsilon'\right)|\Gamma_-, g, s\Big]\right] \\
&+ \mathbf{D}(\lambda B, \lambda\Gamma_-, g, s, \epsilon)\mathcal{X}(B, R, \Gamma_-, g, s, \epsilon) \\
&= [1-\mathbf{D}(B, \Gamma_-, g, s, \epsilon)]\left[1+\frac{1-\delta}{1+r^*}\mathbb{E}\Big[\mathcal{Q}\left(\mathbf{B}'(B, \Gamma_-, g, s, \epsilon), R, \Gamma, g', s', \epsilon'\right)|\Gamma_-, g, s\Big]\right] \\
&+ \mathbf{D}(B, \Gamma_-, g, s, \epsilon)\mathcal{X}(B, R, \Gamma_-, g, s, \epsilon) \\
&= \mathcal{Q}(B, R, \Gamma_-, g, s, \epsilon), \qquad \text{(C.11)}
\end{aligned}
$$

where the second line uses the linearity of $\mathbf{B}'(\cdot)$, the third line uses our guess on $\mathcal{Q}(\cdot)$ and $\mathcal{X}(\cdot)$, the fourth line uses the homogeneity properties of $\mathbf{D}(\cdot)$ and $\mathbf{E}(\cdot)$, and the last line confirms our guess. Again, using equation (20), we can follow the same procedure for $\mathcal{X}(\cdot)$:

$$
\begin{aligned}
\mathcal{X}(\lambda B, R, \lambda\Gamma_-, g, s, \epsilon) &= \frac{1}{1+r^*}\mathbb{E}\Bigg[\theta\mathcal{X}(\lambda B, R, \lambda\Gamma, g', s', \epsilon') + (1-\theta)\left[1-\mathbf{E}(\lambda B, \lambda\Gamma, g', s', \epsilon')\right]\mathcal{X}(\lambda B, R, \lambda\Gamma, g', s', \epsilon') \\
&+ (1-\theta)\mathbf{E}(\lambda B, \lambda\Gamma, g', s', \epsilon')\mathcal{Q}(\kappa\lambda B, \kappa R, \lambda\Gamma, g', s', \epsilon)|\Gamma_-, g, s\Bigg] \\
&= \frac{1}{1+r^*}\mathbb{E}\Bigg[\theta\mathcal{X}(B, R, \Gamma, g', s', \epsilon') + (1-\theta)\left[1-\mathbf{E}(B, \Gamma, g', s', \epsilon')\right]\mathcal{X}(B, R, \Gamma, g', s', \epsilon') \\
&+ (1-\theta)\mathbf{E}(B, \Gamma, g', s', \epsilon')\mathcal{Q}(\kappa B, \kappa R, \Gamma, g', s', \epsilon)|\Gamma_-, g, s\Bigg] \\
&= \mathcal{X}(B, R, \Gamma_-, g, s, \epsilon). \qquad \text{(C.12)}
\end{aligned}
$$

Let's move to (v). Using equations (21) and (22), we can express $\mathcal{R}(\cdot)$ as

$$
\begin{aligned}
\left(\mathcal{R}\left(\lambda N, \lambda B, \lambda\Gamma_-, g, s\right)\right)^{-1} &= \frac{\mathbb{E}\left[\mathcal{Q}\left((1-\delta)\lambda B, +\mathcal{R}\left(\lambda N, \lambda B, \lambda\Gamma_-, g, s\right)\lambda N, \lambda\Gamma, g', s', \epsilon'\right)|\Gamma_-, g, s\right]}{1+r^*} \\
&= \frac{\mathbb{E}\left[\mathcal{Q}\left((1-\delta)\lambda B, +\mathcal{R}\left(N, B, \Gamma_-, g, s\right)\lambda N, \lambda\Gamma, g', s', \epsilon'\right)|\Gamma_-, g, s\right]}{1+r^*} \\
&= \frac{\mathbb{E}\left[\mathcal{Q}\left((1-\delta)B, +\mathcal{R}\left(N, B, \Gamma_-, g, s\right)N, \Gamma, g', s', \epsilon'\right)|\Gamma_-, g, s\right]}{1+r^*} \\
&= \left(\mathcal{R}\left(N, B, \Gamma_-, g, s\right)\right)^{-1} \qquad \text{(C.13)}
\end{aligned}
$$

where the second line uses our guess for $\mathcal{R}(\cdot)$, the third line use the homogeneity of $\mathcal{Q}(\cdot)$, and the last line confirms our guess for $\mathcal{R}(\cdot)$. □

The model normalization from Section 3.2 uses the homogeneity properties just derived. In particular, the detrended model from Section 3.2 comes from setting $\lambda = 1/\Gamma_-$

and using: (i) $v^{nd}(b,g,s,\epsilon) = V^{nd}(\lambda B, \lambda\Gamma_-, g, s, \epsilon)$ and $v^d(b,g,s,\epsilon) = V^d(\lambda B, \lambda\Gamma_-, g, s, \epsilon)$; (ii) $Q(b,g,s,\epsilon) = \mathcal{Q}(\lambda B, 1, \lambda\Gamma_-, g, s, \epsilon)$ and $X(b,g,s,\epsilon) = \mathcal{X}(\lambda B, 1, \lambda\Gamma_-, g, s, \epsilon)$; (iii) $R(n,b,g,s,\epsilon) = \mathcal{R}(\lambda N, \lambda B, \lambda\Gamma_-, g, s, \epsilon)$; and (iv) $\mathbf{d}(b,g,s,\epsilon) = \mathbf{D}(\lambda B, \lambda\Gamma_-, g, s, \epsilon)$ and $\mathbf{e}(b,g,s,\epsilon) = \mathbf{E}(\lambda B, \lambda\Gamma_-, g, s, \epsilon)$; where $b = B/\Gamma_-$ and $n = N/\Gamma_-$.

D Computational algorithm

In this appendix, we provide details about the computation of the detrended model of Section 3.2.

Step 0: We set an evenly spaced grid $\vec{b} = \{b_1, b_2, \ldots, b_{N_b}\}$ for debt service values, and a grid $\vec{\epsilon} = \{\epsilon_1, \epsilon_2, \ldots, \epsilon_{N_\epsilon}\}$ for the transitory shocks to endowment. For Argentina, we use Gauss quadrature nodes for $\vec{\epsilon}$, and we set $N_b = 3000$, $N_\epsilon = 55$, $b_1 = -0.05$, and $b_{N_b} = 0.5$. For Spain, we need a finer grid to deal with converge issues due to longer maturity. of debt. We use the discretization method in Tauchen (1986) with parameter 3.5 for $\vec{\epsilon}$, and we set $N_b = 7350$, $N_\epsilon = 17$, $b_1 = -0.01$, and $b_{N_b} = 0.3$. In both cases, b_1 and b_{N_b} were chosen such that the grid covers all values of b in our simulations.

Step 1: Guess a value for no default $v^{nd}(b,g,s,\epsilon)$ and an implied policy for debt services $\mathbf{b}'(b,g,s,\epsilon)$. We restrict debt service to be on the grid, $\mathbf{b}'(b,g,s,\epsilon) \in \vec{b}$.

Step 2: Given $v^{nd}(b,g,s,\epsilon)$, we compute the value of default $v^d(b,g,s,\epsilon)$ using equation (24). We can then compute the policies for default and re-entry, $\mathbf{d}(b,g,s,\epsilon)$ and $\mathbf{e}(b,g,s,\epsilon)$, as in equations (29) and (30). We also compute the maximum of both values, $w(b,g,s,\epsilon) = \max\left\{v^{nd}(b,g,s,\epsilon),\ v^d(b,g,s,\epsilon)\right\}$.

Step 3: Given $\mathbf{d}(b,g,s,\epsilon)$ and $\mathbf{e}(b,g,s,\epsilon)$, as well as the guess for debt service policy $\mathbf{b}'(b,g,s,\epsilon)$, we jointly solve for prices $Q(b,g,s,\epsilon)$ and $X(b,g,s,\epsilon)$ using equations (27) and (28). We use linear interpolation for $Q(\cdot)$ and $X(\cdot)$ for values of b outside the grid—which may occur because of the recovery κ or the growth rate g, even if the debt service policy is on the grid $\vec{b}$.

Step 4: For each debt service $b_i \in \vec{b}$ on the grid, we compute the expected next-period price of a bond starting with debt service b_i: $Q_i^{\mathbb{E}}(g,s) = \mathbb{E}\left[Q(b_i, g', s', \epsilon')|g,s\right]$. Then, for each $b_i \in \vec{b}$, we use equation (25) to compute the implied rate consistent with $Q_i^{\mathbb{E}}(g,s)$: $R_i(g,s) = \frac{1+r^*}{Q_i^{\mathbb{E}}(g,s)}$. Then, we compute the implied issuance: $n_i(b,g,s) = \frac{b_i-(1-\delta)b}{R_i(g,s)}$. Note that the issuance depends on next-period debt services as well as current debt services.

The above steps yield, for each (b,g,s), a correspondence $\{R_i, n_i\}_i$, resembling the ones in Figures 6 and 8 (where we omitted the inputs of R_i and n_i to ease notation). In what follows, we restrict our attention to the pairs $\{R_i, n_i\}_i$ with implied default probabilities in the next period below 65%.[25]

[25] The default probability restriction is implemented by setting a utility penalty of $1e^{24}$ for the pairs violating the restriction.

For a given n, there may be multiple values of R in the correspondence, so the next step consists of restricting the set of pairs (R_i, n_i) in $\{R_i, n_i\}_i$ that are available to the borrower according to the sunspot realization. In the case of the good sunspot, we simply allow the borrower to choose any of the pairs in the correspondence $\{R_i, n_i\}_i$, because the borrower will always prefer the lowest rates. Therefore, the result is equivalent to computing the increasing interest rate schedule with the lowest rates from which the borrower can choose from.

For the bad sunspot, we first identify the increasing parts of the correspondence. Note that the vector $\vec{b}$ is increasing, so $j > i$ implies $b_j > b_i$, and then $Q_i^{\mathbb{E}} \geq Q_j^{\mathbb{E}}$, and thus $R_i \leq R_j$. We then consider that a pair (R_i, n_i) is in the increasing part of the correspondence if n_{i+1} is larger than n_{i-1}. Then, from the correspondence $\{R_i, n_i\}_i$, we identify the points $\hat{i}$ for which there exists $j > \hat{i}$ such that: $n_j < n_{\hat{i}}$, and n_j is an increasing part of the correspondence. Under the bad sunspot, we make all these $\hat{i}$ point of the correspondence unavailable to the borrower. Thus, this procedure results in an increasing interest rate schedule in which the highest rate is always selected for possible issuance n.

Step 5: For each state (b, g, s), we have a schedule $\{R_i, n_i\}_i$ associated with a next-period debt service b_i in the grid. Given the state (b, g, s, ϵ), we choose b_i to maximize utility as in equation (23). Let $\hat{\mathbf{b}}'(b, g, s, \epsilon)$ be the implied optimal policy. Let $\hat{v}^{nd}(b, g, s, \epsilon)$ be the implied value of no default, and $\hat{w}(b, g, s, \epsilon) = \max\left\{\hat{v}^{nd}(b, g, s, \epsilon),\ v^d(b, g, s, \epsilon)\right\}$ the maximum of both values.

If $\max|w(b, g, s, \epsilon) - \hat{w}(b, g, s, \epsilon)| < 1e^{-5}$, the model converged. Otherwise, we update $v^{nd}(b, g, s, \epsilon)$ and go to **Step 2**.

E Model robustness results

In this appendix, we show how the quantitative results of our model change with reasonable perturbations to the benchmark parameters in Table 2. Specifically, we show both the interest rate schedules in the low-growth state (Figures E.1 and E.2) and simulation results (Tables E.1 and E.2) for Argentina and Spain for different values of the parameters related to the persistence of the low- and high-growth states (p_L and p_H), maturity (δ), recovery rate (κ), standard deviation of the transitory shocks to endowment (σ), cost of default in the high-growth state ($\phi(g_H)$), and discount factor (β).

Maturity δ Columns (2) and (3) in Tables E.1 and E.2 report the simulation results for different values of the parameter δ for the cases of Argentina and Spain, respectively. Their respective interest rate schedules under the benchmark calibration are discussed in Section 5.1 (Figure 7c) and Section 5.2 (Figure 9c). For both Argentina and Spain, the simulation results show that a shorter maturity (higher δ) diminishes the debt dilution incentives, and the borrower is able to issue more debt at lower spreads, associated with lower default rates. In the case of Argentina, by moving δ from 0.15 to 0.60, the average spread decreases from 16.4% to 0.1%, while debt issuance increases from 12% to 29% of GDP. Similarly, for Spain, the average spread decreases from 7.8% to 1.1% and debt issuance increases from 9% to 24% of GDP when δ increases from 0.10 to 0.20.

Persistence of the low growth-state p_L Figures E.1a and E.2a show the interest rate schedules for higher values of p_L (black dashed line) relative to the benchmark (red solid line). In line with the discussion in Section 2, higher values of p_L are associated with an increase in the high interest rates, and the borrower is able to issue less debt. The simulation results in Tables E.1 and E.2 show that, with higher p_L, the borrower chooses endogenous austerity when facing the higher spreads. In the case of Argentina, default rates decrease from 5.1% to 3.6% when p_L increases to 0.65 relative to the benchmark value of 0.60—see columns (1) and (4) in Tables E.1 and E.2.

Recovery rate κ Figures E.1b and E.2b show the interest rate schedules for recovery rates of 70% (black dashed line), lower than the benchmark of 75% (red solid line). In both cases, lower recovery rates increase the high interest rates and the borrower is able to issue less debt. But the changes are quantitatively small. Tables E.1 and E.2 show that the simulation results with lower values of κ are very similar to the benchmark case—see columns (1) and (5) in Tables E.1 and E.2.

Standard deviation of the transitory shocks to endowment σ Figures E.1c and E.2c show the interest rate schedules for higher values of σ (black dashed line) relative to the benchmark (red solid line). In this case, the variation in σ has different implications for the schedules and simulation results between Argentina and Spain. In the case of Argentina, the interest rate schedules shift to the left, so the borrower is able to issue less debt in equilibrium. This is reflected in column (6) in Table E.1, in which average issuance decreases to 22% of GDP in comparison to 25% in the benchmark case in column (1). In the case of Spain, the interest rate schedules shift to the right, so the borrower is able to actually issue more debt in equilibrium. This is reflected in column (6) in Table E.2, in which average issuance decreases to 19% of GDP in comparison to 18% in the benchmark case in column (1). The differences between Argentina and Spain reflects two opposing effects of increasing σ. On the one hand, a higher σ reduces the value of being in default, since smoothing consumption through debt issuance is no longer an option, thus increasing possible debt issuances to the borrower. On the other hand, a higher σ makes bad draws more likely, increasing default probabilities and, thus, lowering possible debt issuances to the borrower. In the case of Argentina, in which the borrower adopts a gambling for redemption strategy, the lower value of repayment dominates, and the borrower is able to issue less debt. In the case of Spain, which adopts an austerity strategy, the lower value of default dominates, and the borrower is able to issue more debt. Overall, the quantitative implications of a change in σ that we analyzed are small, even though we chose values of σ above the 95th percentile of the posterior distribution for most countries in Table 1.

Cost of default in the high-growth state $\phi(g_H)$ Figures E.1d and E.2d show the interest rate schedules for higher values of $\phi(g_H)$ (black dashed line) relative to the benchmark (red solid line) for Argentina and Spain, respectively. Higher values of $\phi(g_H)$ represent a lower cost of default, which makes default relatively more attractive. As a result, for both Argentina and Spain, the interest rate schedules shift to the left, meaning that the borrower can issue less debt in equilibrium. By comparing the simulation results in Tables E.1 and E.2, the results with lower default costs in column (7) are very similar to the benchmark case in column (1).

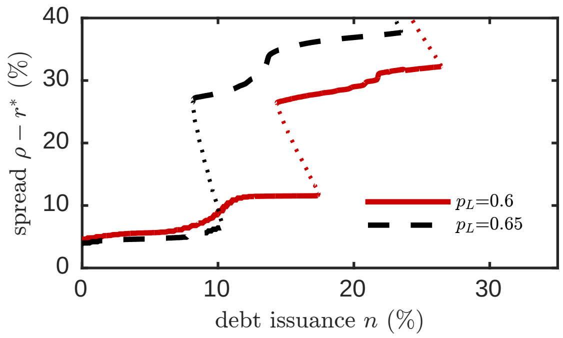
Persistence of the high growth-state p_H Figures E.1e and E.2e show the interest rate schedules for lower values of p_H (black dashed line) relative to the benchmark (red solid line) for Argentina and Spain, respectively. A lower p_H means that the high-growth state is less persistent, so the economy is more likely to enter a stagnation period. This increases the high interest rates in the high-growth schedules, making the gambling for redemption strategy less attractive. With lower incentives to gamble for redemption, and therefore to default, the borrower is able to issue more debt at lower rates. This explains the schedules shifting to the right both in Figures E.1e and E.2e. In the case of Argentina, that actually pushes the borrower to end its gambling for redemption strategy and to adopt austerity instead, so the interest rates become much lower. That is reflected in the simulation results in column (8) in Tables E.1 and E.2. In the case of Argentina, default rates decrease from 5.1% in the benchmark case in column (1) to 0.2% in column (2).

Discount factor β Figures E.1f and E.2f show the interest rate schedules for higher values of β (black dashed line) relative to the benchmark (red solid line) for Argentina and Spain, respectively. A higher value of β means that the borrower is more patient and therefore less likely to adopt a gambling for redemption strategy. Thus, for both for Argentina and Spain, the interest rate schedules shifts to the right with higher β, meaning that the borrower can issue more debt at lower rates. Tables E.1 and E.2 show that default rates are significantly lower with higher β relative to the benchmark—see columns (9) and (1) in Tables E.1 and E.2.

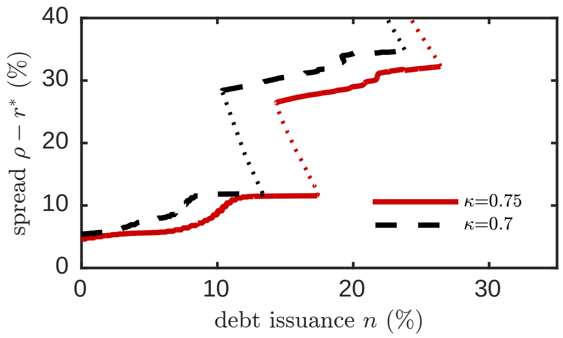
Difference between growth rates ($g_H - g_L$) Figures E.1g and E.2g show the interest rate schedules for different distances between the high- and low-growth rates, which we denote $\Delta^g \equiv g_H - g_L$. We pick g_L and g_H such that, as we vary Δ^g, the (unconditional) average growth rate of the economy remains the same. Therefore, a larger difference Δ^g implies a higher g_H and a lower g_L relative to the benchmark. For Argentina (Figure E.1g) we compare the benchmark with $\Delta^g = 6\%$ (red solid line), to a case with a larger difference in growth rates, $\Delta^g = 8\%$ (black dashed line). For Spain (Figure E.2g), we compare the benchmark with $\Delta^g = 5\%$ (red solid line), to a case with a lower difference in growth rates, $\Delta^g = 4\%$ (black dashed line). The plots show that a higher Δ^g shifts the schedule right for Argentina, while the opposite occurs for Spain. Accordingly, a larger Δ^g increases debt issuance for Argentina while it actually lowers it for Spain– see columns (10) and (1) in Tables E.1 and E.2. The differences between Argentina and Spain reflects two opposing effects of increasing Δ^g. On the one hand, a larger Δ^g implies a more volatile output path as well as a lower g_L, both of which decrease the value of default, thus increasing possible debt issuances to the borrower. One the other hand, a larger Δ^g implies lower output realizations in the low-growth state, which lowers the value of repayment in low-growth periods, thus increasing default rates and lowering possible debt issuances. For Argentina, the reduction in the value of default dominates and the economy is able to issue more debt. For Spain, the increase in the value of repayment dominates. Importantly, the overall changes are quantitatively small, and the plots show that the multiplicity in the interest rate schedules is robust to reasonable changes in Δ^g.

Figure E.1: Interest rate spreads for Argentina in the low growth state (b = 20%): Comparative statics

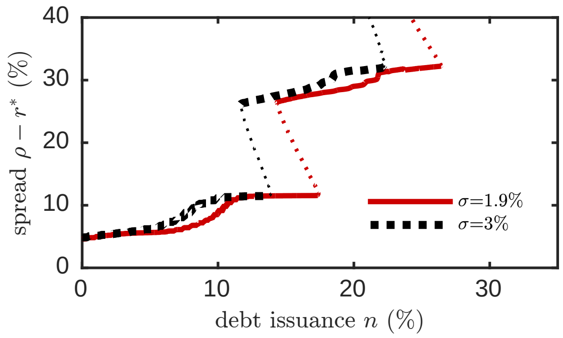
(a) Probability of remaining in low growth (p_L)



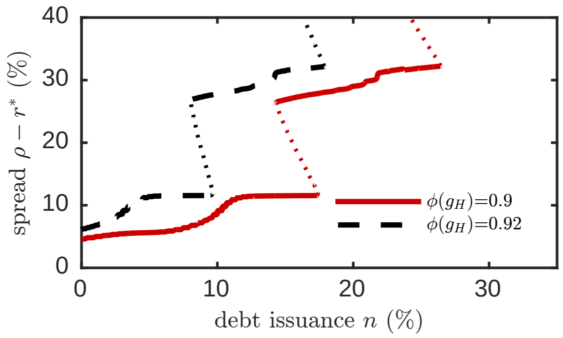
(b) Recovery rate (κ)



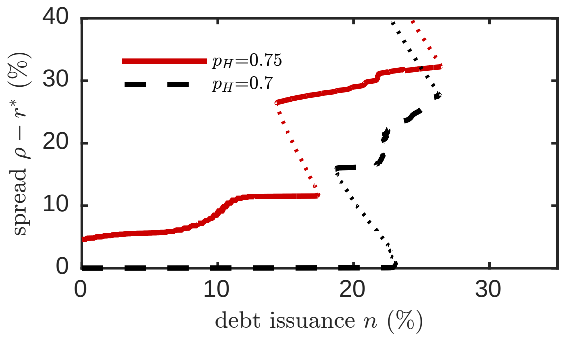
(c) St. deviation of transitory shock (σ)



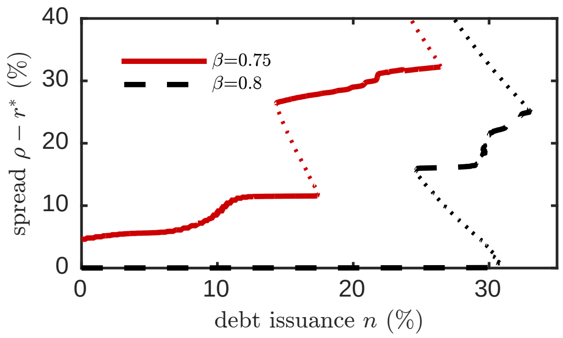
(d) Cost of default in high growth ($\phi(g_H)$)



(e) Probability of remaining in high growth (p_H)



(f) Discount factor (β)



(g) Difference between growth rates ($g_H - g_L$)

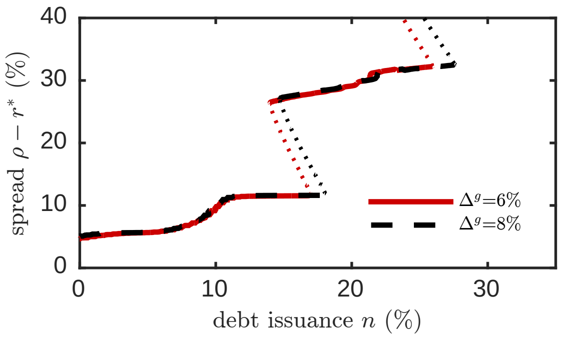
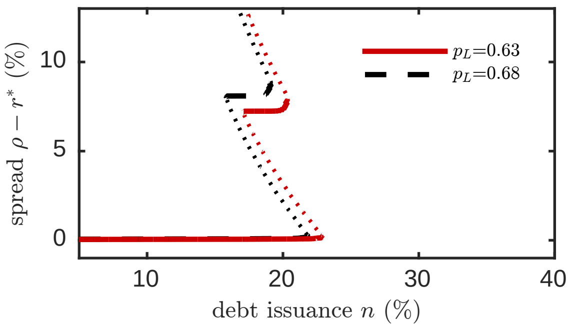
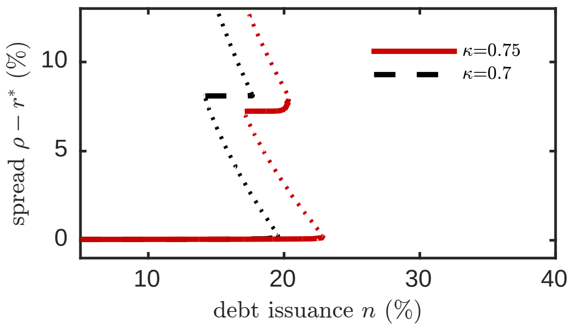


Figure E.2: Interest rate spreads for Spain in the low growth state ($b = 15\%$): Comparative statics

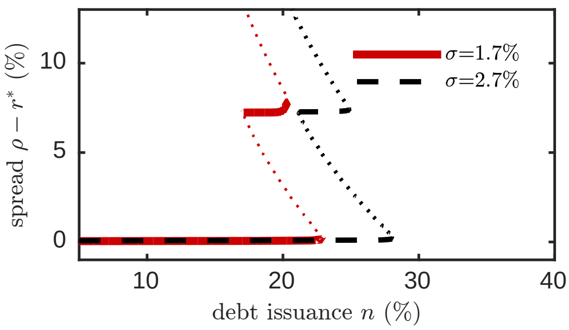
(a) Probability of remaining in low growth (p_L)



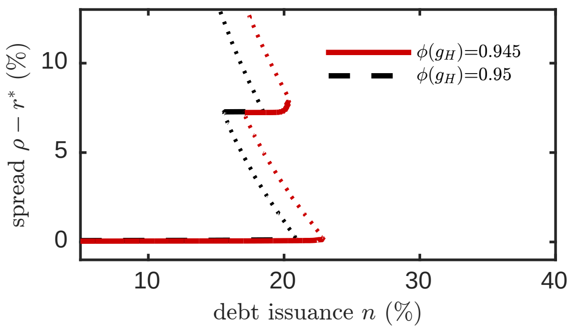
(b) Recovery rate (κ)



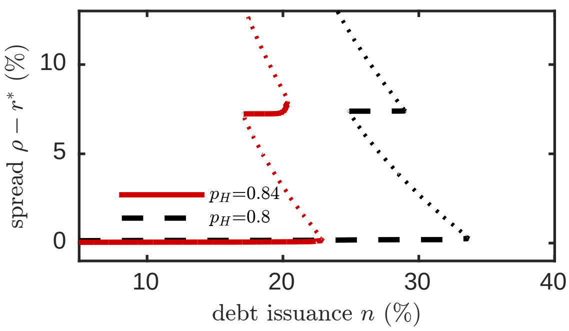
(c) St. deviation of transitory shock (σ)



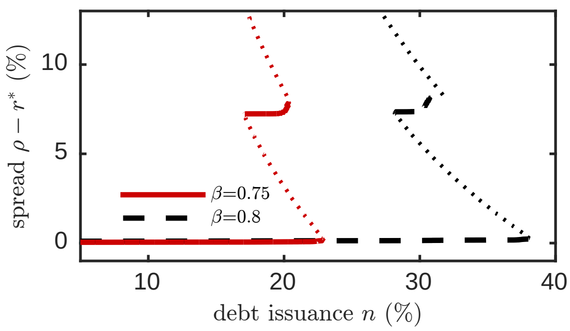
(d) Cost of default in high growth ($\phi(g_H)$)



(e) Probability of remaining in high growth (p_H)



(f) Discount factor (β)



(g) Difference between growth rates ($g_H - g_L$)

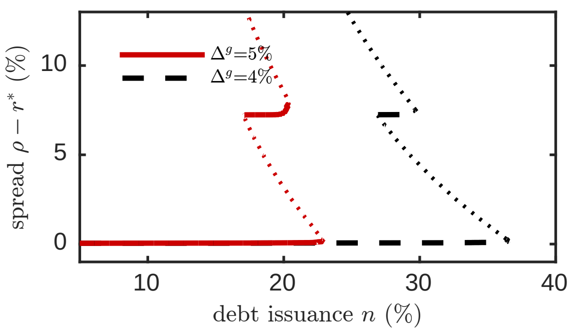


Table E.1: Simulation moments: Argentina

	Benchmark (1)	$\delta = 0.15$ (2)	$\delta = 0.6$ (3)	$p_L = 0.65$ (4)	$\kappa = 0.70$ (5)	$\sigma = 3\%$ (6)	$\phi(g_H) = 0.92$ (7)	$p_H = 0.7$ (8)	$\beta = 0.8$ (9)	$g_H - g_L = 8\%$ (10)
First moments										
avg(b/y)	30	17	31	29	28	27	24	23	26	31
avg($qb/\delta y$)	49	48	50	47	51	43	39	53	61	55
avg(f/y)	50	50	49	49	46	44	40	52	59	52
avg(n/y)	25	12	29	24	24	22	20	21	24	26
avg(tb/y)	5.1	5.2	1.5	5.6	4.4	4.6	4	1.7	1.8	5.2
default rate	5.1	5	0.2	3.6	5.3	5.3	4.9	0.2	0.3	3.8
avg($spread$)	16.4	16.4	0.1	16.7	17.4	16.5	16.5	0.1	0.1	16.5
Low-growth state										
avg(b/y)	22	14	32	18	20	20	18	23	27	23
avg($qb/\delta y$)	11	2	52	25	25	10	6	55	64	23
avg(f/y)	37	39	50	32	33	34	30	53	61	38
avg(n/y)	25	17	28	14	20	23	20	19	22	26
avg(tb/y)	-3	-3.4	4	3.8	0.2	-2.4	-2.7	3.9	4.7	-3.9
default rate	13.3	13	0.6	8.7	13.9	13.7	12.8	0.6	0.8	9.7
avg($spread$)	30.4	24.9	0.1	10.8	31.9	30	30.2	0.1	0.1	30.6
High-growth state										
avg(b/y)	30	18	30	29	28	27	24	22	25	31
avg($qb/\delta y$)	50	50	49	48	53	45	40	52	60	56
avg(f/y)	50	50	48	49	47	45	40	50	58	52
avg(n/y)	25	12	30	24	24	22	20	22	25	26
avg(tb/y)	5.4	5.7	0	5.6	4.6	4.9	4.3	0.1	0.1	5.5
default rate	0	0.1	0	0	0	0.1	0	0	0	0
avg($spread$)	15.9	15.9	0.1	16.7	16.8	15.8	16	0.1	0.1	16
Second moments										
std($spread$) (p.p.)	2.9	2.1	0.1	1.2	3.1	3.4	2.9	0.1	0	3
std(c)/std(y) (p.p.)	2.8	2.6	0.8	3.2	3.2	1.9	2.5	1.5	1.6	2.7

Note: b denotes total debt service (principal+coupon payments), qb/δ denotes the market value of debt, f denotes the face value of debt, n denotes debt issuance, tb denotes trade balance, and y denotes output. We simulate the model economy for 20,000 periods, exclude the first 1,000 periods, and compute moments conditional on not being in default. The default rate is the number of default episodes per 100 periods divided by 100.

Table E.2: Simulation moments: Spain

	Benchmark (1)	$\delta = 0.1$ (2)	$\delta = 0.2$ (3)	$p_L = 0.68$ (4)	$\kappa = 0.70$ (5)	$\sigma = 2.7\%$ (6)	$\phi(g_H) = 0.95$ (7)	$p_H = 0.8$ (8)	$\beta = 0.8$ (9)	$g_H - g_L = 4\%$ (10)
First moments										
avg(b/y)	17	11	23	17	16	18	16	19	20	19
avg($qb/\delta y$)	88	52	98	88	86	95	87	100	110	105
avg(f/y)	86	52	93	85	83	91	84	96	106	100
avg(n/y)	18	9	24	18	18	19	18	20	18	20
avg(tb/y)	-1.4	1.6	-0.9	-1.2	-1.9	-1.5	-1.6	-1.2	1.9	-0.4
default rate	5.2	6	4.7	5.3	5.1	5.1	5.4	5.4	0.4	4.2
avg($spread$)	1.2	7.8	1.1	1.2	1.1	1.1	1.1	1.2	0.2	0.8
Low-growth state										
avg(b/y)	16	10	23	16	15	17	16	18	21	19
avg($qb/\delta y$)	80	33	95	81	77	88	78	95	114	103
avg(f/y)	84	47	93	84	80	90	82	95	110	101
avg(n/y)	17	10	21	16	17	18	17	18	15	18
avg(tb/y)	-0.9	0.2	1.1	-0.1	-2	-0.3	-1	0.5	6	1.2
default rate	18	20	14.7	15.6	17	16.9	17.3	15.4	1.3	14
avg($spread$)	0.1	9.4	0.2	0.2	0.1	0.2	0.2	0.2	0.2	0.1
High-growth state										
avg(b/y)	17	11	23	17	16	18	16	19	20	19
avg($qb/\delta y$)	89	55	99	89	88	97	88	102	109	105
avg(f/y)	86	53	93	85	83	92	84	96	105	100
avg(n/y)	18	9	24	18	18	20	18	21	19	20
avg(tb/y)	-1.5	1.8	-1.4	-1.4	-1.9	-1.8	-1.7	-1.8	0.3	-0.8
default rate	0	0.1	0.4	0	0	0.1	0	0	0	0
avg($spread$)	1.4	7.5	1.4	1.4	1.4	1.3	1.3	1.5	0.2	1
Second moments										
std($spread$) (p.p.)	0.7	1.3	1	0.7	0.7	0.7	0.7	0.9	0	0.8
std(c)/std(y) (p.p.)	2.7	2.3	2.8	2.6	2.9	2.3	2.7	2.8	1.9	2.9

Note: b denotes total debt service (principal+coupon payments), qb/δ denotes the market value of debt, f denotes the face value of debt, n denotes debt issuance, tb denotes trade balance, and y denotes output. We simulate the model economy for 20,000 periods, exclude the first 1,000 periods, and compute the moments conditional on not being in default. The default rate is the number of default episodes per 100 periods divided by 100.